پاسخ تشریحی سوالات دوازدهم دهم A

SUBJECT:

① $f(x+2)=\begin{cases}3x+5 & x\geq 1\\ 4x-1 & x<1\end{cases} \xrightarrow[x=t-2]{x+2=t} f(t)=\begin{cases}3t-1 & t\geq 3\\ 4t-9 & t<3\end{cases}$

$f(3-x)=\begin{cases}3(3-x)-1 & 3-x\geq 3 \quad x\leq 0\\ -3x+8 & \\ 4(3-x)-9 & 3-x<3 \quad x>0\\ -4x+3 & \end{cases}$

$f(x+1)=\begin{cases}3(x+1)-1 & x+1\geq 3 \quad x\geq 2\\ 3x+2 & \\ 4(x+1)-9 & x+1<3 \quad x<2\\ 4x-5 & \end{cases}$

	0		2	
	$-4x+3-4x+5$	$-3x+3-3x+5$		$1-4x+3-3x+2$
	$-8x+8$			$-7x+1$
	$-8x+8\geq 0$	$8+5$		$-5-6-2$
	$x\leq 1$ همواره برقرار	✓ (۱۳)		(-۱۳)
		$-8x+8\geq 0$		$-7x+1\geq 0$
		$x\leq 1$		$7x\geq 1$
				$x\geq \frac{1}{7}$ غ ق ق

$x=(-\infty \quad 1]$

SUBJECT:

(۲) $f(f(x)) < f(x^3)$ $\quad f(x) = (x^3 + x)^3 \rightarrow$ صعودی اکید

$\xrightarrow[f(x)\ \text{اکید}]{\text{صعودی}} f(x) < x^3 \rightarrow (x^3 + x)^3 < x^3$

$\rightarrow x^3 + x < x \rightarrow x^3 < 0 \rightarrow x < 0$

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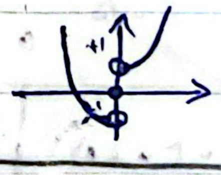
(۳) $y = |x|\left(x^2 + \frac{1}{x}\right)$

$-x^2 - 1$	0	$x^2 + 1$



$\rightarrow$ غیر یکنوا

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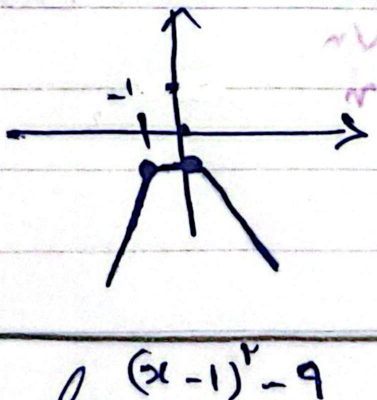
(۴) $f(x) = \dfrac{2x+1}{|x| - |x+1|}$

$\frac{2x+1}{-x+x+1}$	-1	$\frac{2x+1}{-x-x-1}$	0	$\frac{2x+1}{x-x-1}$
$y = 2x+1$		$\frac{2x+1}{-2x-1}$ (−1)		$-2x-1$ (−1)



$\rightarrow$ صعودی در بازه $(-\infty\ \ 0]$

$\boxed{۹۲۰} \leftarrow$

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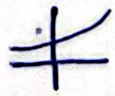
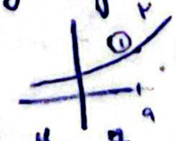
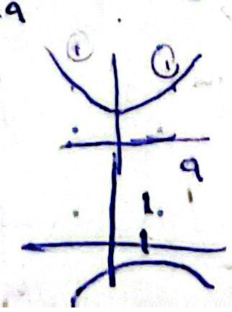
(۵) $y = \log_{2^{-1}} \left((x-1)^2 - 9\right) \rightarrow y = -2\log_2 (|x-1| - 9)$

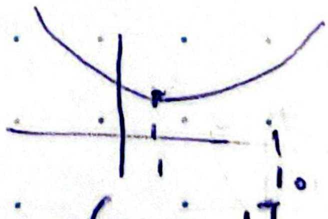
$y = \log_2 x$ $\quad y = \log_2 (x - 9)$





$-2\log_2 (|x-1| - 9)$





$\rightarrow (-\infty\ \ 1]$

SUBJECT:

⑦ $f(x) = (a^2-4)^x$

الف) صعودی الیه ← $a^2-4 > 0$ — $a > 2$

$\rightarrow a > \sqrt{4}$ $a < -\sqrt{4}$

ب) صعودی $a^2-4 \geq 0$ $a \geq \sqrt{4}$ $a \leq -\sqrt{4}$

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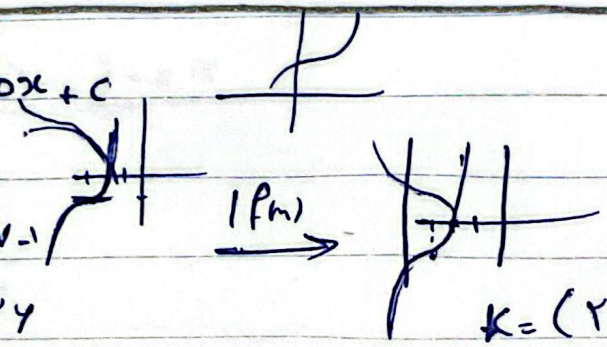
⑧ $f(x) = x^3 + ax^2 + bx + c$

$\rightarrow y = (x+2)^3 - 1$

$\rightarrow y = x^3 + 6x^2 + 12x + 8 - 1$

$y = x^3 + 6x^2 + 12x + 7$



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⑧ $f(f(x)) \leq f(\sqrt{x})$ $f(x) = x + \sqrt{x} - 2$ ← صعودی الیه

$\rightarrow f(x) \leq \sqrt{x}$ $\rightarrow x + \sqrt{x} - 2 \leq \sqrt{x}$

$\rightarrow \boxed{x \leq 2}$ $\rightarrow \{0, 1, 2\}$

$0 \leq x$

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⑨ $9\cos^2 x - 1 \geq 0$ $\cos^2 x \geq \frac{1}{9}$ $\cos x \geq \frac{1}{3}$ $\cos 2x \leq -\frac{1}{3}$

$1 - 9\cos^2 x \geq 0$ $\cos^2 x \leq \frac{1}{9}$, $-\frac{1}{3} \leq \cos x \leq \frac{1}{3}$

$\cos x = \pm \frac{1}{3}$. $\rightarrow 2\cdot 0 = 2^0 = 1$

$\rightarrow$

SUBJECT:

① $f(x) = x - [x+2]$ $\quad g(x) = \frac{2^x - 2^{-x}}{2}$

$R_{g \circ f} = ?$

$f(x) = x - [x] - 2$

$0 \le x - [x] < 1$

$-2 \le x - [x] - 2 < -1$

$g(x) = \frac{2^x - \frac{1}{2^x}}{2} = \frac{2^{2x} - 1}{2^{x+1}}$

$x \to -1$ ⇒ $\frac{2^{-2} - 1}{2^0} = \frac{1}{4} - 1 = -\frac{3}{4}$

$x \to -2$ ⇒ $\frac{2^{-4} - 1}{2^{-1}} = \frac{\frac{1}{16} - 1}{\frac{1}{2}} = \frac{-\frac{15}{16}}{\frac{1}{2}}$

$-\frac{15}{8}$

$\longrightarrow R = [-\frac{15}{8}, -\frac{3}{4})$