حورا_حقیقی

1)

$f(x) = \begin{cases} 2x-1 & x \geq 3 \\ 4x-9 & x < 3 \end{cases}$ $\quad f(3-x) \begin{cases} 5-2x & x \leq 0 \\ 3-4x & x > 0 \end{cases}$

$f(x+1) = \begin{cases} 2x+1 & x \geq 2 \\ 4x-5 & x < 2 \end{cases}$

$f(3-x) - f(x+1) = \begin{cases} 3-4x-2x-1 = -6x+2 & x \geq 2 \\ 3-4x-4x+5 = -8x+8 & 0 < x < 2 \\ 5-2x-4x+5 = -6x+10 & x \leq 0 \end{cases}$

$-6x+2 \geq 0 \quad x \leq \frac{1}{3}$ غ‌ق

$-8x+8 \geq 0 \quad x \leq 1 \Rightarrow 0 < x \leq 1$

$-6x+10 \geq 0 \quad x \leq \frac{10}{6} \Rightarrow x \leq 0$

$D_f = (-\infty, 1]$

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2)

$f(x) = (x^3+x)^3$ صعودی صعودی $\Rightarrow$ اکیداً صعودی

$fof < f(x^3) \Rightarrow x^3 > f(x) \quad x^3 > (x^3+x)^3 \quad x > x^3+x \quad x<0$

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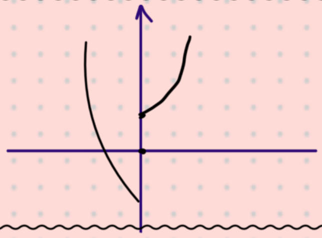
3)

$x \geq 0 \quad (x^3+1)$

$x < 0 \quad (-x^3-1)$



$\Rightarrow$ غیر یکنوا

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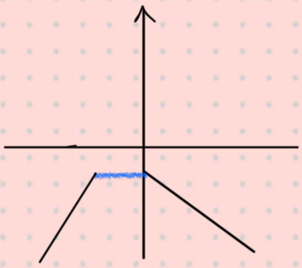
4)

$f(x) = \frac{2x+1}{|x|-|x+1|}$ $\quad 0 \quad -1$

$x \geq 0 \quad -2x-1$

$-1 < x < 0 \quad \frac{2x+1}{-x-x-1} = -1$

$x \leq -1 \quad \frac{2x+1}{-x+x+1} = 2x+1$



$(-1, 0]$ در بازه صعودی است $\Rightarrow a = 0$

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5)

$\log_{\frac{1}{2}}(x^2-2x-8)$ ن ن } در این صورت اکیداً صعودی

① $x^2-2x-8 \quad -\frac{b}{2a} = 1 \Rightarrow (-\infty, 1]$ نزولی $\quad a>0$

② $x^2-2x-8 > 0$

$(x-4)(x+2) > 0 \Rightarrow x < -2 \vee x > 4$

①∩② $\Rightarrow x < -2$

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6)

$(a^2-3)^x \longrightarrow$ اکیداً صعودی $\quad a^2-3 > 1 \quad a^2 > 4 \quad a > 2, \ a < -2$

$(a^2-3)^x \longrightarrow$ صعودی $\quad a^2-3 \geq 1 \quad a^2 \geq 4 \quad a \geq 2, \ a \leq -2$

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7)

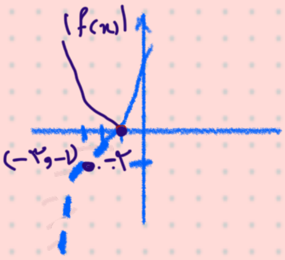
$(-3, -1)$

$(x+3)^3 - 1 = 0$

$(x+3)^3 = 1$

$x+3 = 1$

$x = -2$



$\Rightarrow (k, +\infty) \Rightarrow k = -2$

اکیداً صعودی

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8)

$f(x) = x-2+\sqrt{x}$ صعودی صعودی

$x \geq 0$

$\sqrt{x} \geq f(x)$ $\quad$ شرط لازم $\Rightarrow f(x) \geq 0$

$x-2 \leq 0 \quad x \leq 2$ ②

$x+\sqrt{x}-2 \geq 0$

$(\sqrt{x}+2)(\sqrt{x}-1)$ غ‌ق، 1

غ‌ق $\quad 0 \quad - \quad 1 \quad +$ ①

①∩② $\quad 1 \leq x \leq 2 \vee \{0\} \Rightarrow 1, 2, 0$

۳ عدد

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$\sqrt[2]{9\cos^2 x - 1} \longrightarrow = f(x)$

$\min \cos^2 x = 0 \quad f(x) = \frac{1}{2} - 2 = -\frac{3}{2}$

$\max \cos^2 x = 1 \quad f(x) = 4 - \frac{1}{2} = \frac{15}{2}$

$[a,b] = \left[-\frac{3}{2}, \frac{15}{2}\right] = \frac{15}{4} + \frac{3}{2} = \frac{21}{4}$

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$f(x) = x - [x+2] = \underset{0 \le}{x} - [x] - 2 \quad \boxed{-2 \le f(x) < -1}$

$g(x) = \frac{x^x - x^{-x}}{2}$

$gof(x) = g(f(x)) \quad -2 \le < -1$

$g(-2) = \frac{\frac{1}{4} - 4}{2} = -\frac{15}{8}$

$g(-1) = \frac{\frac{1}{2} - 2}{2} = -\frac{3}{4}$

$R_{gof} = \left[-\frac{15}{8}, -\frac{3}{4}\right)$
