

الف) $y^3 - 2x^2 = 2x - 1 \rightarrow y^3 = 2x^2 + 2x - 1 \xrightarrow{\substack{x=y_1 \\ y=y_2}} \begin{cases} y_1^3 = 2x^2 + 2x - 1 \\ y_2^3 = 2x^2 + 2x - 1 \end{cases} \rightarrow y_1^3 = y_2^3$
 $\xrightarrow{3} y_1 = y_2 \rightarrow$ تابع است

ب) $x \cos y = y \cos x \xrightarrow{x=\frac{\pi}{4}} \frac{\pi}{4} \cos y = y \cos \frac{\pi}{4} \rightarrow y = \frac{\pi}{4} \text{ یا } \frac{3\pi}{4}$ تابع نیست

ج) $\frac{x+y}{y} = \sqrt{xy} \quad x, y > 0 \rightarrow x+y = 2\sqrt{xy} \rightarrow (\sqrt{x} - \sqrt{y})^2 = 0 \rightarrow \sqrt{x} = \sqrt{y} \sim x=y$ تابع است

د) $xy^2 + x^2y - y^3 - y^2 = y^2(x-y) + y^2(x-y) = (x-y)(y^2+y) = 0 \rightarrow x=y$ تابع است
 (هر دو ضلع لایه است)

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الف) $|y| + x^2 = 2y \xrightarrow{y \geq 0} x^2 = y \sim$ تابع است
 $\xrightarrow{y < 0} x^2 = 2y \rightarrow y = \left(\frac{x^2}{2}\right) \sim$ غیر قابل کنترل $\rightarrow$ جمله مثبت یا منفی

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تابع نیست $\rightarrow y \in [0, 1) \rightarrow [x] + [y] = 1 \xrightarrow{x=1} [x] + [y] = 1 \rightarrow [y] = 0 \rightarrow y \in [0, 1)$ د ب

$f(x) = \begin{cases} 2x^2 - b & |x-1| \geq 2 \\ a + 2x & 0 \leq x \leq 3 \end{cases} \xrightarrow{x=3} \begin{cases} 12 - b \\ a + 6 \end{cases} \rightarrow \begin{cases} 12 - b = a + 6 \\ a + b = 12 - 6 = 6 \end{cases}$

$|x-1| \geq 2 \rightarrow x \leq -1 \rightarrow x \leq -1$
 $\sim -x \geq 1 \rightarrow x \leq -1$

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$f(x) = \sqrt{a - bx - x^2} \quad (a \neq 0, b \in \mathbb{Z}), D_f = [b, a], a + b = ?$

$a - bx - x^2 \geq 0 \rightarrow -(x^2 + bx - a) \geq 0 \rightarrow -(x-a)(x-b) \geq 0 \sim a+b = -b \rightarrow$
 $\rightarrow a+b = -1$
 $\begin{matrix} a & b \\ \downarrow & \downarrow \\ -1 & 1 \end{matrix}$
 $a = -ab$

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$y = \frac{p}{\sqrt{ax^2 + bx + c}} \sim \begin{cases} ax^2 + bx + c > 0 \\ \Delta < 0 \end{cases} \rightarrow$ ریشه معادله $\frac{p}{+ \frac{p}{-}}$

$\rightarrow ax^2 + bx + c \xrightarrow{x=0} c = 0 \rightarrow c = -2b \rightarrow 3a + c - 3|b| = b$

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$$y = \sqrt{\frac{3n+2}{n+b} + a} = \sqrt{\frac{3n+2+an+ab}{n+b}}$$

باقیم هرگز دانسته از تیر خارج و $b = -2$ است

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الف) $y = \sqrt{n^3 - n} = \sqrt{n(n^2 - 1)} \quad (n^2 - 1)n \geq 0$

$$\rightarrow [-1, 0] \cup [1, +\infty)$$

ب) $y = \frac{1}{\sqrt{|n|-1}} \sim |n|-1 > 0 \rightarrow |n| > 1 \rightarrow \begin{cases} n > 1 \\ n < -1 \end{cases}$

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الف) $y = 3\sqrt{\frac{1}{|n| - [n]}} \rightarrow |n| - [n] \neq 0 \rightarrow D_y = \mathbb{R} - \mathbb{Z}$

ب) $\sqrt{n} + \sqrt{y} = 9, n \geq 0 \quad \sqrt{y} = 9 - \sqrt{n} \geq 0 \rightarrow 9 \geq \sqrt{n} \rightarrow n \leq 81$

$$\sim 0 \leq n \leq 81$$

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$$y = \frac{1}{\sqrt{[2n - \frac{1}{r}] + [2n + \frac{r}{r} - 1] + 1}} > 0$$

$$\sim 2[2n + \frac{1}{r}] > 0 \rightarrow [2n + \frac{1}{r}] > 0$$

$$2n + \frac{1}{r} > 1 \rightarrow 2n > 1 - \frac{1}{r} \sim n > \frac{1}{r}$$

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$y = \sqrt{1 - \log(rn - a)} \rightarrow rn - a > 0 \rightarrow n > \frac{a}{r}$

$D = (b, c]$

$\rightarrow \frac{a}{r} < n \leq \frac{a+1}{r}, c - b = ? \quad \frac{10+a}{r} - \frac{a}{r} = 10$

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