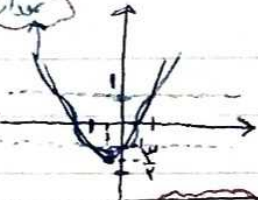


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الف) $y^3 - 2y^2 + 2y - 1$
 $y = \sqrt{2y^2 + 2y - 1}$



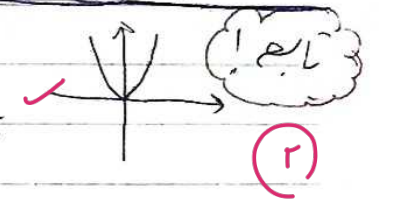
ب) $x(\cos y) = y(\cos x)$
 $x = \frac{\pi}{2} \rightarrow \frac{\pi}{2} \cos y = y \cos \frac{\pi}{2} \rightarrow \frac{\pi}{2} \cos y = 0$

ج) $\frac{x+y}{2} \geq \sqrt{xy}$ $x+y > 0$
 $x+y \geq 2\sqrt{xy} \rightarrow x+y-2\sqrt{xy} = 0 \rightarrow (\sqrt{x}-\sqrt{y})^2 \geq 0$

د) $xy^2 + 2x - y^3 - 2y = 0$
 $x(y^2 + 2) - y(y^2 + 2) = 0$
 $(x-y)(y^2 + 2) = 0 \rightarrow x = y$

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الف) $|y| + x^2 \geq 2y$
 $x^2 - y \geq 2y \rightarrow x^2 \geq 3y$



ب) $[y] + [x] = 1$
 و این نشان می‌دهد که x و y در بازه‌های [0, 1] و [1, 2] قرار می‌گیرند.

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$f(x) = \begin{cases} 2x^2 - b & |x-1| \geq 2 \\ ax^2 & 0 \leq x \leq 3 \end{cases}$
 $a+b=8$
 $a+b \geq 11-b$
 $a+b \geq 11$

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$f(x) = \sqrt{a-bx-x^2}$
 $\{b, a\} \rightarrow$ $a \geq b$
 $a+b \geq 2$
 $a+b \geq 2-1 = 1$

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$y = \frac{r}{\sqrt{a^2 + b^2}}$
 $bf = (-\frac{c}{a}, 2) \rightarrow$
 $a \geq 0$
 $bx + c \geq 0$
 $2b + c \geq 0 \rightarrow c \geq -2b$

$$y = \sqrt{\frac{r^2 + b^2}{nab}} \rightarrow \frac{r^2 + b^2 + nab}{nab}$$

$$\hookrightarrow \frac{(r+a)x + abx}{nab} \rightarrow a = -r \rightarrow \frac{r^2 + b^2}{nab} \rightarrow b > r$$

$$y = \sqrt{x^2 - 2x}$$

$$\hookrightarrow x(x-2) \geq 0 \rightarrow [-2, 0] \cup [2, \infty) \rightarrow D_f$$

$$y = \frac{1}{\sqrt{|x|-1}}$$

$$\rightarrow |x| > 1 \rightarrow x > 1 \rightarrow (R - \{1\}) \rightarrow D_f$$

$$y = \sqrt[3]{\frac{1}{|x| - [x]}} \rightarrow |x| \neq [x] \rightarrow \mathbb{R} - \mathbb{Z}$$

$$y = \sqrt{x} \sqrt{y}$$

$$x \geq 0 \hookrightarrow \sqrt{y} \geq 0 \rightarrow y - \sqrt{y} \geq 0 \rightarrow y \geq \sqrt{y} \rightarrow y \geq 1$$

$$D_f = [\frac{1}{4}, \infty) \rightarrow x \geq \frac{1}{4} \rightarrow y \geq \frac{1}{4} \rightarrow y + \frac{1}{4} \geq 1$$

$$y = \sqrt{\frac{r^2 - \frac{1}{r}}{r}} \rightarrow \frac{r^2 - \frac{1}{r}}{r} \geq 0 \rightarrow r^3 - 1 \geq 0 \rightarrow r \geq 1$$

$$y = \sqrt{1 - \log(r^2 - a)} \rightarrow r^2 > 0 \rightarrow r \geq 0 \rightarrow a = b$$

$$D_f = [b, \infty)$$

Elegance $c = b = 2$

$$\hookrightarrow \Delta = \frac{a}{r} + \frac{a}{r} \geq 2$$

$$1 - \log(r^2 - a) \geq 0 \rightarrow 1 \geq \log(r^2 - a) \rightarrow b \geq r^2 - a$$

$$b + a \geq r^2 \rightarrow \frac{b+a}{2} \geq r \rightarrow \frac{c}{2} \geq r$$