

۲۰

۱۳ در این مسئله: $\frac{1}{2} \cos y$ و $\frac{1}{2} \cos u$

الف) $y'' - r u' = r u - 1 \rightarrow y'' = r u' + r u - 1$
 $\Rightarrow \begin{cases} y_1'' = r u' + r u - 1 \\ y_2'' = r u' + r u - 1 \end{cases} \Rightarrow y_1'' = y_2'' \Rightarrow y_1 = y_2$
نتیجه

ب) $\frac{1}{2} \cos y = \frac{1}{2} \cos u$ ✓
 $y = \frac{\pi}{2} > \frac{\pi}{4}$
 $\frac{\pi}{2} \cos y = \frac{1}{2} \cos \frac{\pi}{4} \rightarrow \frac{\pi}{2} \cos y = 0$

ج) $\frac{u+y}{2} = \sqrt{uy} \rightarrow u+y = 2\sqrt{uy} \rightarrow u+y - 2\sqrt{uy} \rightarrow (\sqrt{u} - \sqrt{y})^2 \rightarrow u=y$ ✓
نتیجه

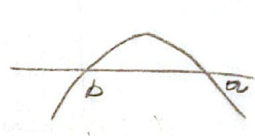
$u y'' + r u - y'' - r y = 0$ ✓
 $y''(u-y) + r(u-y) - (y''+r)(y-y) = 0$
 $\downarrow$
 $u=y$

د) $|y| + u^2 = r y$

$y > 0 \rightarrow y + u^2 = r y \rightarrow u^2 = y \rightarrow \sqrt{y}$ ✓
 $y < 0 \rightarrow u^2 = r y \rightarrow y = \frac{u^2}{r} \rightarrow$ چون $u^2 \geq 0$ و $y < 0$ این معادله
 فقط در صورتی برقرار است که $u=y=0$ باشد.

$\Rightarrow [y] + [u] = 1$ (۱) (۲)
 $\xrightarrow{u=0} [y] = 1 \rightarrow 1 \leq y < 2$ ✓
 در این صورت y مقدار u را مشخص می‌کند.

$f(u) = \begin{cases} r u^2 - b & |u-1| \geq 2 \rightarrow u \geq 3 \text{ یا } u \leq -1 \\ a + r u & 0 \leq u \leq 2 \end{cases}$
 $x=3 \rightarrow 1u-b = a+9 \rightarrow a+b=12$ ✓
نتیجه

$f(u) = \sqrt{-u^2 - bu + a}$ $\frac{b}{-d+b} \frac{a}{-}$ $b \rightarrow -b^2 - b^2 + a = 0 \rightarrow r b^2 = a \rightarrow a=r$ ✓
 $D_f = [b, a]$

 $P = -a = ba \rightarrow b = -1$ ✓
 $a+b=1$ ✓

$y = \frac{r}{\sqrt{a u^2 + b u + c}}$
 $y = \frac{r}{\sqrt{b u + c}}$
 $D = (-\infty, r)$
 $\downarrow$
 $a=0$ ✓
 $b u + c > 0$
 $r a + c - r |b| = 0$ ✓
 $r b + c = 0 \Rightarrow r b = -c$ ✓
نتیجه

$$y = \sqrt{\frac{r_n + r}{n+b}} + a = \sqrt{\frac{r_n + r + an + ab}{n+b}}$$

④ $y = (r, +\infty) \rightarrow$ چون r در مخرج است
 پس y در $(-\infty, 0)$ منتهی است. (r)
 $b = -r$ ✓

الف) $y = \sqrt{u^2 - u}$

$u^2 - u \geq 0 \rightarrow u(u-1) \geq 0$
 $D = [-1, 0] \cup [1, +\infty)$ ✓

$\rightarrow y = \frac{1}{\sqrt{|x|-1}}$

$|x| - 1 > 0 \rightarrow |x| > 1$
 $x > 1 \text{ یا } x < -1$

$D = (-\infty, -1) \cup (1, +\infty)$ ✓

ب) $y = \sqrt{\frac{1}{|x|-x}}$

$|x| - x \neq 0 \rightarrow D = |x| - x$ ✓

$\rightarrow \sqrt{x} + \sqrt{y} = a$

$x \geq 0, \sqrt{y} = a - \sqrt{x} \Rightarrow D = [0, a^2]$ ✓

$\leadsto a - \sqrt{x} \geq 0 \rightarrow a \geq \sqrt{x} \rightarrow x \leq a^2$

$y = \frac{1}{\sqrt{[r_n - \frac{1}{r}] + [r_n + \frac{r}{r}]}}$

$[r_n - \frac{1}{r}] + [r_n + \frac{r}{r}] > 0$
 $[r_n + \frac{r}{r} - 1] + [r_n + \frac{r}{r}] > 0$

$2[r_n + \frac{r}{r}] - 1 > 0 \rightarrow [r_n + \frac{r}{r}] > \frac{1}{2}$

$\left[u > \frac{1}{2} \right] \Rightarrow u + \frac{r}{r} \geq 1$ ✓

$\frac{1}{r} \rightarrow [r_n + \frac{r}{r}] > \frac{1}{r}$
 $\downarrow$
 $r_n + \frac{r}{r} \geq r$

$y = \sqrt{1 - (y^{r_n - a})} \quad D = (b, c)$

① $r_n - a > 0 \rightarrow x > \frac{a}{r}$

② $1 \geq y^{r_n - a}$

$\rightarrow \frac{10}{r} \geq r_n - a$
 $\frac{10+a}{r} \geq x$

$I \cap II \Rightarrow$

$\left(\frac{a}{r} \right) < x \leq \frac{10+a}{r}$

$c - b = \frac{10+a}{r} - \frac{a}{r} = \frac{10}{r}$ ✓

(r) ⑤