

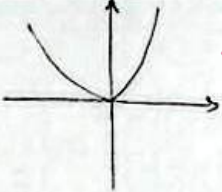
الف) $y^3 - 2x^2 = 2x - 1$ تعریف کنیم $y_1^3 = 2x_1^2 + 2x_1 - 1$ $y_2^3 = 2x_2^2 + 2x_2 - 1$ $\Rightarrow y_1 = y_2$ تابع هست. $\checkmark$

(۲)

ب) $x \cos y = y \cos x \Rightarrow$ مثال نقض $x = \frac{\pi}{2} \Rightarrow \frac{\pi}{2} \cos y = y \cos \frac{\pi}{2} \Rightarrow \cos y = 0 \Rightarrow y = \{2k\pi \pm \frac{\pi}{2}\}$
 $\Rightarrow y = \{\frac{\pi}{2}, \frac{3\pi}{2}, \dots\} \rightarrow$ تابع نیست. $\checkmark$

ج) $\frac{x+y}{y} = \sqrt{xy} \quad x, y > 0 \Rightarrow x+y = 2\sqrt{xy} \Rightarrow x+y - 2\sqrt{xy} = 0 \Rightarrow (\sqrt{x} - \sqrt{y})^2 = 0$
 $\Rightarrow \sqrt{x} = \sqrt{y} \Rightarrow x = y$ تابع هست. $\checkmark$

د) $xy^2 + 2x - y^2 - 2y = 0 \Rightarrow y^2(x-y) + 2(x-y) = 0 \Rightarrow (x-y)(y^2+2) = 0 \Rightarrow x=y$ تابع هست. $\checkmark$

ه) $|y| + x^2 = 2y \Rightarrow \begin{cases} y \geq 0 : y = x^2 \\ y < 0 : y = \frac{x^2}{2} \end{cases}$ $\checkmark$ تابع است. 

(۲)

ب) $[y] + [x] = 1 \Rightarrow$ مثال نقض $x=0 \Rightarrow [y] = 1 \Rightarrow 1 \leq y < 2$ تابع نیست. $\checkmark$

پ) $f(x) = \begin{cases} 2x^2 - b & |x-1| \geq 2 \\ a + 2x & 0 \leq x \leq 3 \end{cases} \Rightarrow \begin{cases} 2x^2 - b & x \geq 3 \\ a + 2x & 0 \leq x \leq 3 \\ 2x^2 - b & x \leq -1 \end{cases} \xrightarrow{x=3} 18 - b = a + 6 \Rightarrow a + b = 12$ $\checkmark$

(۲)

الف) a, b صحیح $\Rightarrow a+b = \frac{-(-b)}{-1} = -b \Rightarrow a = -2b$
 $ab = \frac{a}{-1} = -a \Rightarrow b = -1$ $\checkmark \Rightarrow a = 2$ $\checkmark \Rightarrow a+b = 2-1 = 1$ $\checkmark$

(۲)

ب) $0 = (-\infty, 2)$ $\Rightarrow a=0$ $x=2 \Rightarrow 2b+c=0 \Rightarrow c=-2b$ $\checkmark$
 $\Rightarrow 3a+c-3|b| = 0 \Rightarrow -2b-3(-b) = b$ $\checkmark$

(۲)

ج) $y = \sqrt{\frac{2x^2+y}{x+b}} + a = \sqrt{\frac{(3+a)x+y+ba}{x+b}} \Rightarrow D = (2, +\infty)$ $\Rightarrow 3+a=0 \Rightarrow a=-3$ $\checkmark$
 $2+b=0 \Rightarrow b=-2$ $\checkmark$

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الف) $y = \sqrt{x^3 - x} \Rightarrow x^3 - x \geq 0 \Rightarrow x(x^2 - 1) \geq 0 \Rightarrow x(x-1)(x+1) \geq 0 \Rightarrow D = [-1, 0] \cup [1, +\infty)$ $\checkmark$

ب) $y = \frac{1}{\sqrt{|x|-1}} \Rightarrow |x|-1 > 0 \Rightarrow |x| > 1 \Rightarrow x > 1, x < -1 \Rightarrow D = \mathbb{R} - [-1, 1]$ $\checkmark$

(۲)

الف) $y = \sqrt[r]{\frac{1}{|x| - [x]}} \Rightarrow |x| \neq [x] \Rightarrow D = \mathbb{R} - \mathbb{Z}$ ✓

(r) -1

ب) $\sqrt{x} + \sqrt{y} = 4 \Rightarrow 1) x \geq 0$

2) $\sqrt{y} = 4 - \sqrt{x} \Rightarrow 4 - \sqrt{x} \geq 0 \Rightarrow \sqrt{x} \leq 4 \Rightarrow x \leq 16$

$\Rightarrow D = [0, 16]$ ✓

$y = \frac{1}{\sqrt{[rx - \frac{1}{r}] + [rx + \frac{r}{r}]}}$

$\Rightarrow [rx - \frac{1}{r}] + [rx + \frac{r}{r}] > 0 \xrightarrow{+r} [rx - \frac{1}{r} + r] + [rx + \frac{r}{r}] - r > 0 \Rightarrow r[rx + \frac{r}{r}] > r$ (r) -9
 $\Rightarrow [rx + \frac{r}{r}] > 1 \Rightarrow r \leq rx + \frac{r}{r} \Rightarrow rx \geq \frac{1}{r} \Rightarrow x \geq \frac{1}{r}$ ✓ $D = [\frac{1}{r}, +\infty)$

$y = \sqrt{1 - \lg(rx - a)}$

1) $rx - a > 0 \Rightarrow x > \frac{a}{r}$

2) $1 - \lg(rx - a) \geq 0 \Rightarrow \lg(rx - a) \leq 1 \Rightarrow rx - a \leq 10 \Rightarrow x \leq \frac{a+10}{r}$

$\Rightarrow D = (\frac{a}{r}, \frac{a+10}{r}]$ (r) -10

$\Rightarrow b = \frac{a}{r} > c = \frac{a+10}{r} \Rightarrow c - b = \frac{a+10}{r} - \frac{a}{r} = \frac{10}{r}$ ✓