

۱-

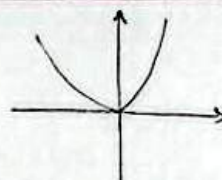
الف) $y^3 - 2x^2 = 2x - 1 \xrightarrow{\text{تعریف متغیر}} \begin{cases} y_1^3 = 2x_1^2 + 2x_1 - 1 \\ y_2^3 = 2x_2^2 + 2x_2 - 1 \end{cases} \xrightarrow{x_1 = x_2} y_1^3 = y_2^3 \Rightarrow y_1 = y_2$ تابع هست.

ب) $x \cos y = y \cos x \Rightarrow$ مورد نقیض $x = \frac{\pi}{2} \Rightarrow \frac{\pi}{2} \cos y = y \cos \frac{\pi}{2} = 0 \Rightarrow \cos y = 0 \Rightarrow y = \{2k\pi \pm \frac{\pi}{2}\}$
 $\Rightarrow y = \{\frac{\pi}{2}, \frac{3\pi}{2}, \dots\} \rightarrow$ تابع نیست.

ج) $\frac{x+y}{2} = \sqrt{xy} \quad x, y > 0 \Rightarrow x+y = 2\sqrt{xy} \Rightarrow x+y - 2\sqrt{xy} = 0 \Rightarrow (\sqrt{x} - \sqrt{y})^2 = 0$
 $\Rightarrow \sqrt{x} = \sqrt{y} \Rightarrow x = y$ تابع هست.

د) $xy^2 + 2x - y^2 - 2y = 0 \Rightarrow y^2(x-y) + 2(x-y) = 0 \Rightarrow (x-y)(\underbrace{y^2+2}_{\text{همیشه بزرگتر از ۰}}) = 0 \Rightarrow x = y$ تابع هست.

۲-

الف) $|y| + x^2 = 2y \Rightarrow \begin{cases} y \geq 0 : y = x^2 \\ y < 0 : y = \frac{x^2}{3} \end{cases}$  تابع است.

ب) $[y] + [x] = 1 \Rightarrow$ مورد نقیض $x = 0 \Rightarrow [y] = 1 \Rightarrow 1 \leq y < 2$ تابع نیست.

۳-

$$f(x) = \begin{cases} 2x^2 - b & |x-1| \geq 2 \\ a + 2x & 0 \leq x \leq 3 \end{cases} \Rightarrow \begin{cases} 2x^2 - b & x \geq 3 \\ a + 2x & 0 \leq x \leq 3 \\ 2x^2 - b & x \leq -1 \end{cases} \xrightarrow{x=3} 18 - b = a + 6 \Rightarrow a + b = 12$$

۴-

مورد نقیض $a, b \Rightarrow a + b = \frac{-(-b)}{-1} = -b \Rightarrow a = -2b$
 $ab = \frac{a}{-1} = -a \Rightarrow b = -1 \Rightarrow a = 2 \Rightarrow a + b = 2 - 1 = 1$

۵-

$0 = (-\infty, 2)$ $\rightarrow$ مورد نقیض $a = 0 \xrightarrow{x=2} 2b + c = 0 \Rightarrow c = -2b$
سیستم با دو معادله سه مجهول $\Rightarrow 3a + c - 3|b| = 0 \Rightarrow -2b - 3(-b) = b$

۶-

$y = \sqrt{\frac{2x^2 + 2}{x+b}} + a = \sqrt{\frac{(2+a)x + 2 + ba}{x+b}} \Rightarrow D = (2, +\infty) \Rightarrow \begin{cases} 2+a=0 \Rightarrow a=-2 \\ 2+b=0 \Rightarrow b=-2 \end{cases}$ که برای زوج است.

الف) $y = \sqrt{x^3 - x} \Rightarrow x^3 - x \geq 0 \Rightarrow x(x^2 - 1) \geq 0 \Rightarrow x(x-1)(x+1) \geq 0 \Rightarrow \frac{-1}{-1} \leq x \leq \frac{1}{-1} \Rightarrow D = [-1, 0] \cup [1, +\infty)$

ب) $y = \frac{1}{\sqrt{|x|-1}} \Rightarrow |x|-1 > 0 \Rightarrow |x| > 1 \Rightarrow x > 1, x < -1 \Rightarrow D = \mathbb{R} - [-1, 1]$

الف) $y = \sqrt[r]{\frac{1}{|x| - [x]}} \Rightarrow |x| \neq [x] \Rightarrow D = \mathbb{R} - \mathbb{Z}$

-8

ب) $\sqrt{x} + \sqrt{y} = 4 \Rightarrow 1) x \geq 0$

2) $\sqrt{y} = 4 - \sqrt{x} \Rightarrow 4 - \sqrt{x} \geq 0 \Rightarrow \sqrt{x} \leq 4 \Rightarrow x \leq 16$

$\Rightarrow D = [0, 16]$

$y = \frac{1}{\sqrt{[rx - \frac{1}{r}] + [rx + \frac{r}{r}]}}$

$\Rightarrow [rx - \frac{1}{r}] + [rx + \frac{r}{r}] > 0 \xrightarrow{+r} [rx - \frac{1}{r} + r] + [rx + \frac{r}{r}] - r > 0 \Rightarrow r[rx + \frac{r}{r}] > r$

$\Rightarrow [rx + \frac{r}{r}] > 1 \Rightarrow r \leq rx + \frac{r}{r} \Rightarrow rx \geq \frac{1}{r} \Rightarrow x \geq \frac{1}{r} \quad D = [\frac{1}{r}, +\infty)$

-9

$y = \sqrt{1 - \lg(rx - a)}$

1) $rx - a > 0 \Rightarrow x > \frac{a}{r}$

2) $1 - \lg(rx - a) \geq 0 \Rightarrow \lg(rx - a) \leq 1 \Rightarrow rx - a \leq 10 \Rightarrow x \leq \frac{a+10}{r} \Rightarrow D = (\frac{a}{r}, \frac{a+10}{r}]$

-10

$\Rightarrow b = \frac{a}{r} > c = \frac{a+10}{r} \Rightarrow c - b = \frac{a+10}{r} - \frac{a}{r} = \underline{10}$