

الف) $y'' - 2u' = 2u - 1 \rightarrow y'' = 2u' + 2u - 1$

(1)

$$\left. \begin{aligned} y_1'' &= 2u' + 2u - 1 \\ y_2'' &= 2u' + 2u - 1 \end{aligned} \right\} \rightarrow y_1'' = y_2'' \rightarrow y_1 = y_2 \text{ تابع همت}$$

ب) $u \cos y = y \sin u$

فرض $u = \frac{\pi}{2}$, $\cos u = 0 \Rightarrow \cos y = 0 \Rightarrow y = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$ تابع نیست

ب) $\frac{u+y}{2} = \sqrt{uy}$ $u, y > 0$

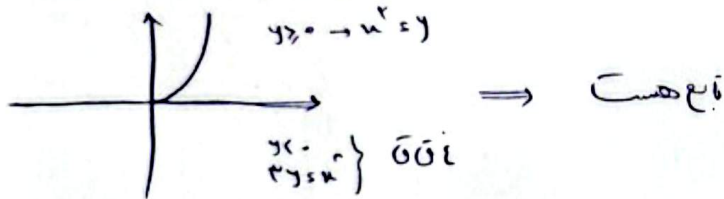
$u+y = 2\sqrt{uy} \rightarrow u+y - 2\sqrt{uy} = 0 \Rightarrow (\sqrt{u} - \sqrt{y})^2 = 0 \Rightarrow \sqrt{u} = \sqrt{y} \rightarrow u = y$ تابع همت

د) $uy' + 2u - y'' - 2y = 0$

$y'' - uy' + 2y - 2u = 0 \rightarrow y''(y-u) + 2(y-u) = 0 \Rightarrow (y-u)(y''+2) = 0 \Rightarrow y = u$ تابع همت

الف) $|y| + u^2 = 2y \rightarrow 2y - |y| = u^2$

(2)



ب) $[y] + [u] = 1$ $u=0, [y]=1 \Rightarrow y = 1, 2, 3, \dots$ تابع نیست

$$f(u) = \begin{cases} u^2 - b & |u-1| > 2 \rightarrow -2 > u-1 > 2 \rightarrow -1 > u > 3 \\ a + 2u & 0 \leq u \leq 2 \end{cases}$$

(3)

$u=2, 4+a = 12-b \rightarrow a+b = 12$

$a, b, a \Rightarrow a - ab - a^2 = 0 \Rightarrow 2b^2 - 2b^2 - 4b^2 = 0 \Rightarrow 4b^2 + 2b^2 - 2b^2 = 0$
 $\Rightarrow 2b^2(2b^2 + b - 1) = 0$
 $\Rightarrow b = 0, \frac{a+1}{a+1} \Rightarrow b = -1$
 $\Rightarrow a+b = 2-1 = 1$
 $b = -\frac{1}{2} \Rightarrow b \in \mathbb{Z} \Rightarrow \text{تابع همت}$

(4)

$y = \frac{2}{\sqrt{ax^2 + bx + c}}$ $a \leq 0$
 $b < 0$

(5)

$\frac{0}{2} = 2 \Rightarrow 2b + c = 0 \Rightarrow c = -2b$

$2(0) + c - 2|b| = -2b + 2b = 0$

$$y = \sqrt{\frac{ru+r}{u+b}} + a$$

(7)

$$\frac{ru+r}{u+b} + a = \frac{ru+r+au+ab}{u+b} = \frac{u(r+a)+r+ab}{u+b} \rightarrow r+a=0 \Rightarrow a=-r$$

$$\xrightarrow{u \rightarrow r} \xrightarrow{u \rightarrow b} r+b=0 \Rightarrow \boxed{b=-r}$$

$$\text{ii)} y = \sqrt{u^2 - u}$$

$$u(u-1)(u+1)$$

$$\begin{array}{c} -1 \quad 0 \quad 1 \\ - \quad + \quad - \quad + \end{array}$$

$$\rightarrow D_y = [-1, 0] \cup [1, +\infty)$$

(8)

$$\text{b)} y = \frac{1}{\sqrt{|u|-1}} \Rightarrow |u|-1 > 0 \Rightarrow |u| > 1 \Rightarrow -1 < u < 1$$

$$D_y = (-\infty, -1) \cup (1, +\infty)$$

$$\text{iii)} y = \sqrt{\frac{1}{|u| - [u]}}$$

$$|u| - [u] \neq 0 \Rightarrow |u| \neq [u]$$

$$D_y = \mathbb{R} - \mathbb{W}$$

(9)

$$\text{b)} \sqrt{u} + \sqrt{y} = 9 \Rightarrow \sqrt{y} = 9 - \sqrt{u}$$

$$\text{① } u \geq 0$$

$$\text{② } 9 - \sqrt{u} \geq 0 \Rightarrow \sqrt{u} \leq 9 \Rightarrow 0 \leq u \leq 81 \} \xrightarrow{\text{①, ②}} D_y = [0, 81]$$

$$y = \frac{1}{\sqrt{[ru - \frac{1}{r}] + [ru + \frac{c}{r}]}}$$

$$[ru - \frac{1}{r}] + [ru + \frac{c}{r}] > 0$$

$$\rightarrow [ru - \frac{1}{r}] + [ru + \frac{1}{r} + 1] > 0 \rightarrow [ru - \frac{1}{r}] + [ru + \frac{1}{r}] + 1 > 0$$

$$\rightarrow [ru + \frac{1}{r}] + [ru + \frac{1}{r}] > 0$$

$$\rightarrow r[ru + \frac{1}{r}] > 0 \Rightarrow [ru + \frac{1}{r}] > 0 \Rightarrow ru + \frac{1}{r} \geq 1 \Rightarrow ru \geq \frac{1}{r} \Rightarrow u \geq \frac{1}{r}$$

$$\rightarrow D_y = [\frac{1}{r}, +\infty)$$

(10)

$$y = \sqrt{1 - \lg(ru - a)} \Rightarrow 1 - \lg(ru - a) \geq 0 \rightarrow \lg(ru - a) \leq 1$$

(11)

$$\rightarrow ru - a \leq 1 \Rightarrow u \leq \frac{a}{r} + \frac{1}{r} \quad \text{①}$$

$$\text{② } ru - a > 0 \Rightarrow u > \frac{a}{r}$$

$$\left. \begin{array}{l} \text{①} \\ \text{②} \end{array} \right\} \rightarrow \frac{a}{r} < u \leq \frac{a}{r} + \frac{1}{r}$$

$$c \cdot b = \frac{a}{r} + \frac{1}{r} - \frac{a}{r} = \boxed{\frac{1}{r}}$$