

تابع f و g را به صورت $f(x) = x^2 + 2x - 1$ و $g(x) = x^2 + 2x - 1$ در نظر بگیرید.
 الف) $g(x) - f(x) = x^2 + 2x - 1 - (x^2 + 2x - 1) = 0 \Rightarrow x^2 + 2x - 1 = x^2 + 2x - 1 \Rightarrow x^2 + 2x - 1 = x^2 + 2x - 1$
 ب) $\cos y = y \cos x \Rightarrow \cos y = y \cos \frac{\pi}{4} \Rightarrow \cos y = \frac{y}{\sqrt{2}} \Rightarrow \cos y = \frac{y}{\sqrt{2}}$
 ج) $\frac{x+y}{x-y} = \sqrt{xy} \Rightarrow x+y = \sqrt{xy}(x-y) \Rightarrow x+y - \sqrt{xy}(x-y) = 0 \Rightarrow (\sqrt{x}-\sqrt{y})^2 = 0 \Rightarrow \sqrt{x} = \sqrt{y} \Rightarrow x = y$
 د) $x^2 + 2x - y^2 = 0 \Rightarrow x^2 + 2x = y^2 \Rightarrow (x+1)^2 - 1 = y^2 \Rightarrow (x+1)^2 - y^2 = 1 \Rightarrow (x+1-y)(x+1+y) = 1$
 از این دو معادله می توان $x = y$ را به دست آورد.

کدام یک از روابط زیر تابع است؟
 الف) $|y| + x^2 = 2xy$
 ب) $[y] + [x] = 1$
 ج) $y = x^2$
 د) $y = \frac{x^2}{x^2+1}$
 پاسخ: ج) $y = x^2$
 برای $x=1 \Rightarrow [y] + [1] = 1 \Rightarrow [y] = 0 \Rightarrow y \in [0, 1)$
 پس تابع نیست.

تابع $f(x) = \begin{cases} x^2 - b & |x-1| \geq 2 \\ a + 2x & 0 < |x-1| < 2 \end{cases}$
 الف) $|x-1| \geq 2 \Rightarrow x \leq -1 \text{ یا } x \geq 3$
 ب) $|x-1| < 2 \Rightarrow -1 < x < 3$
 ج) $f(x) = \begin{cases} x^2 - b & x \leq -1 \text{ یا } x \geq 3 \\ a + 2x & -1 < x < 3 \end{cases}$
 د) $f(2) = 4 - b = a + 4 \Rightarrow a - b = 0 \Rightarrow a = b$
 ه) $a + b = 12$

تابع $f(x) = \sqrt{a - bx - x^2}$
 الف) $f(x) = \sqrt{-x^2 - bx + a}$
 ب) $p = \frac{c}{a} = \frac{a}{-1} = -a \Rightarrow axb = -a \Rightarrow b = -1$
 ج) $s = \frac{-b}{a} = \frac{-(-b)}{-1} = -b \Rightarrow a + b = -b \Rightarrow a - 1 = 1 \Rightarrow a = 2$
 د) $a + b = 2 - 1 = 1$

تابع $y = \frac{2}{\sqrt{ax^2 + bx + c}}$
 الف) $ax^2 + bx + c > 0$
 ب) $a = 0 \Rightarrow bx + c > 0 \Rightarrow x > -\frac{c}{b}$
 ج) $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 د) $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 ه) $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(۶) ؟ $\lim_{x \rightarrow +\infty} \frac{x^2 + 1}{x + b} = \frac{x^2 + 1 + a}{x + b}$ $\lim_{x \rightarrow +\infty} \frac{x^2 + 1}{x + b} = \frac{x^2 + 1 + a}{x + b}$ $\frac{x^2 + 1 + a}{x + b} = \frac{x(x + a) + 1 + ab}{x + b}$ و $D_f = (x, +\infty) \Rightarrow x + b \neq 0 \Rightarrow x - (-b) = 0$ $\boxed{b = -1} \Rightarrow b = -1$ و $x + a = 0 \Rightarrow x = -a$ و $a \neq -b$	۶
الف) $y = \sqrt{x^2 - 1}$ $x(x^2 - 1) \geq 0 \Rightarrow x < 0, x = 1, x = -1$ $\frac{-1}{-1} + \frac{1}{1} = 0$ $D_f = [-1, 0] \cup [1, +\infty)$ ب) $y = \frac{1}{\sqrt{ x - 1}} \Rightarrow x - 1 > 0 \Rightarrow x > 1$ (I) $x > 1$ (II) $x < -1$ $D_f = (-\infty, -1) \cup (1, +\infty)$	۷
الف) $y = \sqrt{\frac{1}{ x - [x]}}$ $\Rightarrow x - [x] \neq 0 \Rightarrow x \neq [x]$ $D_f = \mathbb{R} - \mathbb{Z}$ ب) $\sqrt{x} + \sqrt{y} = 4 \Rightarrow \sqrt{y} = 4 - \sqrt{x} \Rightarrow 4 - \sqrt{x} \geq 0 \Rightarrow \sqrt{x} \leq 4 \Rightarrow x \leq 16$ $\sqrt{x} \geq 0 \Rightarrow x \geq 0 \Rightarrow x \in [0, 16]$	۸
$x + \frac{1}{x} = (x - \frac{1}{x}) + 2$ $y = \sqrt{(x - \frac{1}{x}) + (x + \frac{1}{x})} > 0 \Rightarrow [x - \frac{1}{x}] + [(x + \frac{1}{x}) + 2] \rightarrow [x - \frac{1}{x}] + [x + \frac{1}{x}] + 2$ $\Rightarrow \sqrt{2[x - \frac{1}{x}] + 2} > 0 \Rightarrow 2[x - \frac{1}{x}] > -2 \Rightarrow [x - \frac{1}{x}] > -1 \Rightarrow [x - \frac{1}{x}] \geq 0$ $\Rightarrow x - \frac{1}{x} \geq 0 \Rightarrow x \geq \frac{1}{x} \Rightarrow x \geq \frac{1}{x} \Rightarrow D_f = [\frac{1}{x}, +\infty)$	
الف) $x - a > 0 \Rightarrow x > a \Rightarrow x > \frac{a}{1} \rightarrow b$ ب) $1 - \log(x - a) > 0 \Rightarrow \log(x - a) < 1 \Rightarrow x - a < 10 \Rightarrow x < a + 10 \Rightarrow x < \frac{a + 10}{1}$ $c - b \Rightarrow \frac{a + 10}{1} - \frac{a}{1} \Rightarrow \frac{a + 10 - a}{1} = \frac{10}{1} = 10$	۱۰