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$$x = \begin{cases} 3y-1 & y \geq 0 \\ -y^2 & y < 0 \end{cases}$$

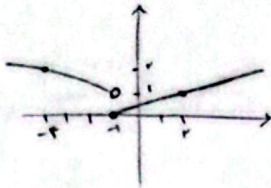
$$x = 3y-1 \rightarrow y = \frac{x+1}{3} \text{ از } \frac{x+1}{3} \geq 0 \rightarrow x \geq -1$$

$$x = -y^2 \rightarrow y = \pm \sqrt{-x} \text{ از } \sqrt{-x} \geq 0 \rightarrow -x \geq 0 \rightarrow x \leq 0$$

چون از یک بهینه است

$$\rightarrow y = \begin{cases} \frac{x+1}{3} & x \geq -1 \\ \sqrt{-x} & x < -1 \end{cases}$$

تابع به شکل تابع هست



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$$g(x) = \sqrt{\frac{x f(x-2)}{f(x)}} \rightarrow \frac{x f(x-2)}{f(x)} \geq 0$$

$$f(x-2) = 0 \rightarrow x \in \{-2, 5, 7\}$$

$$f(x) = 0 \rightarrow x \in \{-4, 2, 5\}$$

$$x \geq 0 \rightarrow x \geq 0$$

Sign chart for $\frac{x f(x-2)}{f(x)}$:

-4	-2	0	2	5*	7
-	+	-	+	-	+

$D_g = (-4, -2) \cup (0, 2) \cup (5, 7)$

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$$f(x) = \frac{2x^2 - 3x + 2}{x^2 - 4x}$$

الف) $D_f(x)$? $x^2 - 4x \neq 0 \rightarrow x(x-4) \neq 0 \rightarrow x \neq 0, 4, -2 \rightarrow D_f = \mathbb{R} - \{0, 4, -2\}$

ب) $D_{\sqrt{f}} = ?$

$$\frac{2x^2 - 3x + 2}{x(x-4)(x+2)} \geq 0 \rightarrow x(x-4)(x+2) > 0$$

Sign chart for $x(x-4)(x+2)$:

-2	0	4
-	+	-

$D_{\sqrt{f}} = (-2, 0) \cup (4, +\infty)$

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الف) $y = \sqrt{\frac{x-2}{x^2 - 2x^2 - 1}}$

$$\frac{x-2}{x^2 - 2x^2 - 1} \geq 0 \rightarrow \frac{(x-2)}{(x-2)(x+2)} \geq 0$$

Sign chart for $\frac{x-2}{(x-2)(x+2)}$:

-2	2
-	+

$D = (-2, +\infty) - \{2\}$

ب) $y = \frac{2x^2 + 1}{x-2[-x]}$

$$x-2[-x] = 0 \rightarrow x+2x = 0 \rightarrow 3x = 0 \rightarrow x = 0$$

Sign chart for $x-2[-x]$:

0
-

$D = \mathbb{R} - \{0\}$

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الف) $f(x) = \sqrt{\frac{x(x^2-1)}{|x|+x}}$

$$\frac{x(x^2-1)}{|x|+x} \geq 0 \rightarrow \frac{-1 \cdot 0 \cdot 1}{-+ -+ -+} \rightarrow [1, 0] \cup [1, +\infty) \text{ ①}$$

ب) $f(x) = \sqrt{\frac{|x-2| - x^2}{|x|-1}}$

$$\frac{|x-2| - x^2}{|x|-1} \geq 0$$

Sign chart for $\frac{|x-2| - x^2}{|x|-1}$:

-2	1
-	+

$D_f = [-2, 1]$

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الف) $y = \sqrt{(x-2)[x-3]}$

$$(x-2)[x-3] \geq 0$$

Sign chart for $(x-2)[x-3]$:

2	3
-	+

$D = [2, 3]$

ب) $y = \sqrt{|x-1|[x-3]}$

$$|x-1|[x-3] \geq 0 \rightarrow [x-3] \geq 0 \rightarrow x-3 \geq 0 \rightarrow x \geq 3$$

Sign chart for $|x-1|[x-3]$:

3
-

$D = [3, +\infty)$

$$y = \frac{1}{|n-2|-1-k}$$

$$|n-2|-1-k \geq 0 \rightarrow |n-2|-1 = k \quad (*)$$

$$\begin{aligned} |n-2|-1 = k &\rightarrow |n-2| = k+1 \rightarrow \begin{cases} n-2 = k+1 \rightarrow n = k+3 \\ n-2 = -k-1 \rightarrow n = 1-k \end{cases} \\ |n-2|-1 = -k &\rightarrow |n-2| = 1-k \rightarrow \begin{cases} n-2 = 1-k \rightarrow n = 3-k \\ n-2 = k-1 \rightarrow n = 1+k \end{cases} \end{aligned}$$

نکته: تنها ۳ عدد صحیح در کانون نامیده باید در مقدار از مقادیر به دست آمده بالا باشد و باید به (*) مقدار k مثبت است.

$$k+3 \geq 1-k \rightarrow k \geq -1 \rightarrow \text{غیر قابل قبول چون } k \text{ منفی است}$$

$$k+3 = 3-k \rightarrow k \geq 0 \rightarrow n = 3, n = 1 \rightarrow \text{تفاوت}$$

$$k+3 \geq 1+k \rightarrow \text{مطلوبه}$$

$$1-k \geq 1+k \rightarrow k \geq 0 \rightarrow n = 3, n = 1 \rightarrow \text{تفاوت}$$

$$3-k \geq k+1 \rightarrow k \geq 1 \rightarrow n = 2, n = 0, n = 2 \rightarrow \text{این مقدار قابل قبول است}$$

$$f(n) = \frac{\frac{n^2+3n+2}{n^2+3n+2}}{\frac{n+1}{n-1}}$$

$$\left. \begin{aligned} n-1 \neq 0 &\rightarrow n \neq 1 \\ \frac{n+1}{n-1} \neq 0 &\rightarrow n \neq -1 \\ n^2+3n+2 \neq 0 &\rightarrow n \neq -2, -1 \end{aligned} \right\} n = 3$$

$$n^2-2nn+3n \geq 0 \xrightarrow{n=3} n^2-4n+9=0 \rightarrow (n-3)^2=0 \rightarrow \boxed{\text{تنها یک ریشه ی مضاعف به ازای } n=3}$$

$$y = \log_{n+1} \log_n \frac{1}{x}$$

$$\begin{aligned} n &> 0 \\ n &\neq 1 \\ n+1 &> 0 \rightarrow n > -1 \\ n+1 &\neq 0 \rightarrow n \neq -1 \end{aligned}$$

$$\log_n \frac{1}{x} > 0 \xrightarrow{n > 1} \frac{1}{x} > 1 \quad \times$$

$$\xrightarrow{0 < n < 1} \frac{1}{x} < 1 \quad \checkmark \text{ مقبول}$$

$$\rightarrow \boxed{D_f(0,1)}$$

$$D_{f(n)} = [-2, 3]$$

$$D_{f(n^2-1)} = ?$$

$$-2 \leq n^2-1 \leq 3 \rightarrow -1 \leq n^2 \leq 4 \rightarrow \frac{-4}{n^2} \leq \frac{1}{n^2} \leq \frac{4}{n^2} \rightarrow \frac{1}{n^2} \leq \frac{4}{n^2} \rightarrow |n| \leq \frac{2}{\sqrt{4}}$$

$$\boxed{-\frac{2}{\sqrt{4}} \leq n \leq \frac{2}{\sqrt{4}}}$$