

عالی ریاضیات

مجموعه مقدماتی

$$f(n) = \begin{cases} 2n-1 & [n] > \tau = n \geq \omega \\ \tau n + 3 & [n] \leq \tau = n < \omega \end{cases}$$

1

$$n \geq \omega \leadsto g = \sqrt{2n-1 - n+1} \leadsto g = \sqrt{n} \leadsto n \geq 0$$

$$\leadsto n \geq \omega \wedge n \geq 0 \leadsto [\omega, +\infty) \textcircled{1}$$

$$n < \omega \leadsto g = \sqrt{\tau n + 3 - n + 1} \leadsto g = \sqrt{3n + 4} \leadsto n \geq -\frac{4}{3}$$

$$\leadsto n < \omega \wedge n \geq -\frac{4}{3} \leadsto [-\frac{4}{3}, \omega) \textcircled{2}$$

$$\textcircled{1} \cup \textcircled{2} \leadsto [-\frac{4}{3}, \omega) \cup [\omega, +\infty) = [-\frac{4}{3}, +\infty) \checkmark$$

$$g = \frac{(2n\sqrt{3} + \sqrt{3})^3}{(\tau n^2 + \tau n + 1)^2} = \frac{\sqrt{3}(2n+1)}{(2n+1)^2} = \frac{\sqrt{3}}{2n+1}$$

$\leadsto n \neq -\frac{1}{2} \textcircled{1}$

$$\frac{\sqrt{3}}{2n+1} \xrightarrow{\textcircled{1}} D = \mathbb{R} - \left\{ -\frac{1}{2} \right\}$$

$$f(n) = \begin{cases} \frac{n^3 + 2n^2 - n - 2}{n^2 - 1} & n \neq \pm 1 \\ \frac{an+b}{n+c} & n = \pm 1 \end{cases}$$

2

$$n=2 \rightarrow 2+c = \frac{1+1-2-2}{2} \rightarrow 2+c = 2 \rightarrow c=0 \quad (1)$$

$$n=1 \rightarrow 1+c = \frac{a+2}{1+b} \rightarrow 3 = \frac{a+2}{1+b} \rightarrow 3+3b = a+2 \rightarrow a=3b+1 \quad (2)$$

$$n=-1 \rightarrow -1+2 = \frac{-a+2}{b-1} \rightarrow 1 = \frac{-(3b+1)+2}{b-1} \rightarrow b-1 = -3b-1+2$$

$$\rightarrow b-1 = -3b+1 \rightarrow 4b=2 \rightarrow b=\frac{1}{2} \quad (3)$$

$$b+c \stackrel{(1),(3)}{\rightarrow} 2+\frac{1}{2} = \frac{5}{2} = 2.5 \quad \square$$

$$g(n) = \frac{n^2-1}{n^2+an+b}$$

$$f(n) = \frac{n+1}{n-1} \quad (2)$$

$$\rightarrow g(n) = \frac{(n-1)(n+1)}{n^2+an+b}$$

$$\rightarrow \frac{(n-1)(n+1)}{(n-1)^2} = \frac{n+1}{n-1}$$

دو تابع در سمت راست برابرند و ضرایب برابر باشند

ضرایب برابر پس برای برابر بودن ضرایب باید برابر باشند $\rightarrow c=1$

$$n^2+an+b = (n-1)^2 \rightarrow n^2+an+b = n^2-2n+1 \rightarrow a=-2, b=1 \quad (3)$$

$$\stackrel{(3),(2)}{\rightarrow} a+b+c = -2+1+1 = 0 \quad \square$$

$$f(n) = (n-3)^2(n-1) \rightarrow f(n+1) = \underbrace{((n+1)-3)^2}_{n-2} \underbrace{((n+1)-1)}_n \quad (2)$$

$$\rightarrow f(n+1) = (n^2-4n+1)n \quad (1)$$

$$\rightarrow f(1-n) = \underbrace{((1-n)-3)^2}_{-n-2} \underbrace{((1-n)-1)}_{-n}$$

$$\leadsto f(1-x) = (x^2 + 1x + 1)(-x) \leadsto f(1-x) = (-x^2 - 1x - 1)x \quad \textcircled{v}$$

$$g(x) = f(x+1) + f(1-x) \xrightarrow{\textcircled{v} \wedge \textcircled{v}} (x^2 - 1x + 1)x + (-x^2 - 1x - 1)x$$

$$\leadsto g(x) = x(\cancel{x^2 - 1x + 1} - \cancel{x^2 - 1x - 1}) \leadsto g(x) = x(-1x) \leadsto$$

$$g(x) = -1x^2 \quad \checkmark$$

$$f(1-2x) = \begin{cases} \sqrt[3]{1-2x} + 1 & x \leq 0 \\ 1 - \sqrt{1-2x} & x > 0 \end{cases}$$

$$f(-1) + f(3) = ?$$

② $\leadsto$

$$f(-1) \leadsto -1 = 1 - 2x \leadsto x = 1 \xrightarrow{x > 0} f(-1) = 1 - \sqrt{1} = 0 \quad \checkmark \textcircled{v}$$

$$f(3) \leadsto 3 = 1 - 2x \leadsto x = -1 \xrightarrow{x \leq 0} f(3) = -1 + 1 = 0 \quad \checkmark \textcircled{v}$$

$$f(-1) + f(3) \xrightarrow{\textcircled{v} \wedge \textcircled{v}} 0 + 0 = 0 \quad \checkmark$$

$$f(x + \sqrt{x}) = x^2 + 3x$$

$$f(4) = ?$$

② $\leadsto$

$$x + \sqrt{x} = 4 \leadsto x + \sqrt{x} - 4 = 0 \leadsto (\sqrt{x} - 1)(\sqrt{x} + 4) \leadsto \sqrt{x} = 1, -4$$

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$$\leadsto \sqrt{x} = 1 \leadsto x = 1 \quad \checkmark \textcircled{v}$$

$$f(4) = x^2 + 3x \xrightarrow{\textcircled{v}} f(4) = \underbrace{1^2 + 12}_{13} \leadsto f(4) = 13 \quad \checkmark$$

$$f(x) = x^w + x = x(x^w + 1)$$

$$f(x-1) + f(1-x) = ?$$

(7) $\leftarrow$ $\wedge$

$$\left. \begin{array}{l} x-1 = x' \\ 1-x = -x' \end{array} \right\} \rightarrow \underbrace{f(x-1) + f(1-x)}_{x'^w + x'} = x'^w + x' - x'^w - x' = 0 \rightarrow$$

$$f(x-1) + f(1-x) = 0 \quad \square$$

$$f(x) = \sqrt{kx - \frac{w}{k}a} + k - 1$$

$$g_k(x) = \sqrt{b - \frac{w}{k}x} + \frac{w}{k}$$

(1) $\leftarrow$ $\wedge$

$$\left. \begin{array}{l} D_f \rightarrow kx - \frac{w}{k}a \geq 0 \rightarrow x \geq \frac{w}{k}a \\ D_{g_k} \rightarrow b - \frac{w}{k}x \geq 0 \rightarrow x \leq \frac{b}{\frac{w}{k}} \end{array} \right\} \rightarrow D_f \cap D_{g_k} = \{1\} \rightarrow$$

پس در این دو دامنه، $x=1$ را می‌گیریم

$$\begin{aligned} x=1 &\rightarrow f(1) = \sqrt{k - \frac{w}{k}a} + k - 1 \rightarrow \frac{w}{k}f(1) = \frac{w}{k}\sqrt{k - \frac{w}{k}a} + \frac{w}{k}k - \frac{w}{k} \quad (1) \\ &\rightarrow g_k(1) = \sqrt{b - \frac{w}{k}} + \frac{w}{k} \quad (2) \end{aligned}$$

$$\frac{w}{k}f(1) - g_k(1) = 0 \quad (1) \quad (2) \rightarrow \frac{w}{k}\sqrt{k - \frac{w}{k}a} + \frac{w}{k}k - \frac{w}{k} - \sqrt{b - \frac{w}{k}} - \frac{w}{k} = 0$$

$$\rightarrow \frac{w}{k}\sqrt{k - \frac{w}{k}a} - \sqrt{b - \frac{w}{k}} = \frac{w}{k} \quad (3) \rightarrow 0 = \frac{w}{k}k + \frac{w}{k} \rightarrow k = -1 \quad \left. \begin{array}{l} k - \frac{w}{k}a = 0 \rightarrow a = \frac{k}{\frac{w}{k}} \\ b - \frac{w}{k} = 0 \rightarrow b = \frac{w}{\frac{w}{k}} \end{array} \right\}$$

$$\rightarrow \frac{ka}{\frac{w}{k}} - \frac{b}{\frac{w}{k}} - k = k - \frac{w}{k} + 1 = \frac{w}{k} \quad \square$$

$$f(x) = \sqrt{m-x} + x \quad g_k(x) = \sqrt{xk+x}$$

(2) $\leftarrow$ 10

$$D_{f \times g_k} = [-1, v] \leadsto D_f \cap D_{g_k} = [-1, v] \textcircled{1}$$

$$D_f \ni m-x \geq 0 \leadsto x \leq m \leadsto D_f = (-\infty, m]$$

$$D_{g_k} \ni xk+x \geq 0 \leadsto x \geq -1 \leadsto D_{g_k} = [-1, +\infty)$$

$$\left. \begin{array}{l} D_f \cap D_{g_k} = [-1, m] \\ \textcircled{1} \end{array} \right\} \begin{array}{l} \rightarrow D_f \cap D_{g_k} = [-1, m] \\ \leadsto m = v \textcircled{2} \end{array}$$

$$\rightarrow \sqrt{\underbrace{xk+x}_x} = x\sqrt{k}$$

$$\underbrace{f(v) - g_k(v)}_{\rightarrow \sqrt{v-x} + x} = 0\sqrt{k} \leadsto v+x - x\sqrt{k} = 0\sqrt{k} \leadsto v+x = \sqrt{k} \leadsto x = \sqrt{k} - v \textcircled{3}$$

$$\textcircled{2} \wedge \textcircled{3} \leadsto v + \sqrt{k} - v = 0 + \sqrt{k}$$

$$\boxed{0 + \sqrt{k}}$$