

← $\sqrt[n]{a}$ $\sqrt[n]{b}$

$$f(n) = \begin{cases} \sqrt[n]{n-1} & [n] > \sqrt[n]{n} = n \geq \omega \\ \sqrt[n]{n+1} & [n] \leq \sqrt[n]{n} = n < \omega \end{cases}$$

← 1

$$n \geq \omega \leadsto g = \sqrt[n]{\sqrt[n]{n-1} - n + 1} \leadsto g = \sqrt[n]{n} \leadsto n \geq 0 \\ \leadsto n \geq \omega \wedge n \geq 0 \leadsto [\omega, +\infty) \textcircled{1}$$

$$n < \omega \leadsto g = \sqrt[n]{\sqrt[n]{n+1} - n + 1} \leadsto g = \sqrt[n]{\sqrt[n]{n+1}} \leadsto n \geq \frac{-1}{2} \\ \leadsto n < \omega \wedge n \geq \frac{-1}{2} \leadsto [-\frac{1}{2}, \omega) \textcircled{2}$$

$$\textcircled{2} \cup \textcircled{1} \leadsto [-\frac{1}{2}, \omega) \cup [\omega, +\infty) = [-\frac{1}{2}, +\infty)$$

$$g = \frac{\sqrt[n]{\sqrt[n]{n^r + \sqrt[n]{n+1}} + \sqrt[n]{n}}}{(\sqrt[n]{n^r + \sqrt[n]{n+1}})^r} = \frac{\sqrt[n]{n}(\sqrt[n]{n+1})}{(\sqrt[n]{n+1})^r} = \frac{\sqrt[n]{n}}{(\sqrt[n]{n+1})^{r-1}} \\ \leadsto n \neq \frac{-1}{r} \textcircled{1}$$

← 2

$$\frac{r}{\sqrt[n]{n+1}} \xrightarrow{\textcircled{1}} D = \mathbb{R} - \{-\frac{1}{r}\}$$

$$f(n) = \begin{cases} \frac{n^r + \sqrt[n]{n} - n - r}{n^r - 1} & n \neq \pm 1 \\ \frac{an + r}{n + b} & n = \pm 1 \end{cases}$$

← 3

$$n=r \rightarrow r+c = \frac{\cancel{A} + \cancel{A} - r - r}{\cancel{w}} \rightarrow r+c = r \rightarrow c=r \quad (1)$$

$$n=1 \rightarrow 1+c = \frac{a+r}{1+b} \rightarrow w = \frac{a+r}{1+b} \rightarrow w+wb = a+r$$

$$\rightarrow a = wb+1 \quad (2)$$

$$n=-1 \rightarrow -1+r = \frac{-a+r}{b-1} \xrightarrow{(2)} 1 = \frac{-(wb+1)+r}{b-1} \rightarrow b-1 = -wb-1+r$$

$$\rightarrow b-1 = -wb-1 \rightarrow rb = r \rightarrow b = \frac{1}{r} \quad (3)$$

$$b+c \xrightarrow{(1),(3)} r + \frac{1}{r} = \frac{a}{r} = r, a$$

$$g(n) = \frac{n^r-1}{n^r+an+b}$$

$$f(n) = \frac{n+1}{n-c}$$

$$\rightarrow g(n) = \frac{(n-1)(n+1)}{n^r+an+b}$$

$$\xrightarrow{(1)} \frac{(n-1)(n+1)}{(n-1)^r} = \frac{n+1}{n-1}$$

① تابع در سرانجام برابر است و ضابطه در انتهای برابر است

ضابطه ها برابر است برای
برابر بودن ضابطه ها باید c یک باشد

بسیار دانه هر r R است $c=1$

$$n^r+an+b = (n-1)^r \rightarrow n^r+an+b = n^r-rn+1 \rightarrow a = -r, b=1 \quad (3)$$

$$\xrightarrow{(3),(2)} a+b+c = -r+1+1=0$$

$$f(n) = (n-r)^r(n-1) \rightarrow f(n+1) = \left(\underbrace{(n+1)-r}_{n-r} \right)^r \left(\underbrace{(n+1)-1}_n \right)$$

$$\rightarrow f(n+1) = (n^r - rn + r^2)n \quad (1)$$

$$\rightarrow f(1-n) = \left(\underbrace{(1-n)-r}_{-n-r} \right)^r \left(\underbrace{(1-n)-1}_{-n} \right)$$

$$\leadsto f(1-x) = (x^2 + 1x + 1)(-x) \leadsto$$

$$f(1-x) = (-x^2 - 1x - 1)x \quad \textcircled{v}$$

$$g(x) = f(x+1) + f(1-x) \xrightarrow{\textcircled{v} \wedge \textcircled{v}} (x^2 - 1x + 1)x + (-x^2 - 1x - 1)x$$

$$\leadsto g(x) = x(\cancel{x^2 - 1x + 1} - \cancel{x^2 - 1x - 1}) \leadsto g(x) = x(-1x) \leadsto$$

$-1x$

$$g(x) = -1x^2$$

$$f(1-2x) = \begin{cases} \sqrt[3]{1-2x} + 1 & x \leq 0 \\ 1 - \sqrt{1-2x} & x > 0 \end{cases}$$

$$f(-1) + f(3) = ?$$

$$f(-1) \leadsto -1 = 1 - 2x \leadsto x = 1 \xrightarrow{x > 0} f(-1) = 1 - \sqrt{1} = 0 \quad \textcircled{v}$$

$$f(3) \leadsto 3 = 1 - 2x \leadsto x = -1 \xrightarrow{x \leq 0} f(3) = -1 + 1 = 0 \quad \textcircled{v}$$

$\sqrt[3]{1-2x} + 1$

$$f(-1) + f(3) \xrightarrow{\textcircled{v} \wedge \textcircled{v}} 0 + 0 = 0$$

$$f(x + \sqrt{x}) = x^2 + 3x$$

$$f(4) = ?$$

$$x + \sqrt{x} = 4 \leadsto x + \sqrt{x} - 4 = 0 \leadsto (\sqrt{x} - 1)(\sqrt{x} + 4) \leadsto \sqrt{x} = 1, -4$$

$\swarrow \searrow$

$$\leadsto \sqrt{x} = 1 \leadsto x = 1 \quad \textcircled{v}$$

$$f(4) = x^2 + 3x \xrightarrow{\textcircled{v}} f(4) = \underbrace{1^2 + 12}_{13} \leadsto f(4) = 13$$

$$f(x) = x^w + x = x(x^w + 1)$$

$$f(x-1) + f(1-x) = ?$$

↪

$$\left. \begin{array}{l} x-1 = x' \\ 1-x = -x' \end{array} \right\} \rightarrow \underbrace{f(x-1) + f(1-x)}_{x'^w + x'} = x'^w + x' - x'^w - x' = 0 \rightarrow$$

$$f(x-1) + f(1-x) = 0$$

$$f(x) = \sqrt{t-x-va} + k-1$$

$$g_k(x) = \sqrt{b-t-x} + vk$$

↪

$$\left. \begin{array}{l} D_f \rightarrow t-x-va \geq 0 \rightarrow x \leq \frac{t}{v} - \frac{va}{v} \\ D_{g_k} \rightarrow b-t-x \geq 0 \rightarrow x \leq \frac{b-t}{v} \end{array} \right\} \rightarrow D_f \cap D_{g_k} = \{1\} \rightarrow$$

⊙
در هر دو طرف ضرب کنیم و به دست آوریم

$$x=1 \rightarrow f(1) = \sqrt{t-va} + k-1 \rightarrow v f(1) = v\sqrt{t-va} + vk-v \quad \textcircled{v}$$

$$\rightarrow g_k(1) = \sqrt{b-t} + vk \quad \textcircled{v}$$

$$v f(1) - g_k(1) = k \quad \textcircled{v} \textcircled{v} \rightarrow v\sqrt{t-va} + vk-v - \sqrt{b-t} - vk = k$$

$$\rightarrow v\sqrt{t-va} - \sqrt{b-t} = vk+v \quad \textcircled{v} \rightarrow 0 = vk+v \rightarrow k = -1$$

$$\left. \begin{array}{l} t-va = 0 \rightarrow a = \frac{t}{v} \\ b-t = 0 \rightarrow b = t \end{array} \right\}$$

$$\rightarrow \frac{va}{v} - \frac{b}{v} - k = t-v+1 = v$$

$$f(x) = \sqrt{m-x} + x \quad g_k(x) = \sqrt{xk+x}$$

q. 10

$$D_{f \times g_k} = [-1, v] \leadsto D_f \cap D_{g_k} = [-1, v] \text{ (1)}$$

$$D_f \leadsto m-x \geq 0 \leadsto x \leq m \leadsto D_f = (-\infty, m]$$

$$D_{g_k} \leadsto xk+x \geq 0 \leadsto x \geq -1 \leadsto D_{g_k} = [-1, +\infty)$$

$$\left. \begin{array}{l} \text{(1)} \\ \text{(2)} \end{array} \right\} \begin{array}{l} D_f \cap D_{g_k} = [-1, m] \\ \text{or } m = v \text{ (2)} \end{array}$$

$$\rightarrow \sqrt{\underbrace{xk+x}_x} = x\sqrt{k}$$

$$\underbrace{f(v)} - \underbrace{g_k(v)} = 9\sqrt{v} \leadsto v + v - v\sqrt{v} = 9\sqrt{v} \leadsto v + v = 10\sqrt{v} \leadsto v = 10\sqrt{v} - v \text{ (3)}$$

$$\rightarrow \underbrace{\sqrt{v-v} + v}_v$$

$$\text{(3)} \wedge \text{(3)}$$

$$\leadsto v + 10\sqrt{v} - v = 10 + 10\sqrt{v}$$