

$$f(x) - x + 1 \geq 0 \rightarrow x \leq f(x) + 1$$

$$f(x) = \begin{cases} 2x-1 & x \geq 0 \\ x+1 & x < 0 \end{cases} \rightarrow \begin{aligned} & \text{شرایط ۱: } x \geq 0 \rightarrow x \leq 2x-1+1 \rightarrow x \leq 2x \rightarrow x \geq 0 \\ & \text{شرایط ۲: } x < 0 \rightarrow x \leq x+1+1 \rightarrow x \leq x+2 \rightarrow x \geq -2 \end{aligned}$$

$$D = (1) \cap (2) : \quad \begin{array}{c} 0 \\ -2 \end{array} \Rightarrow D = [-2, +\infty)$$

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$$y = \frac{2x^2 + 2x + 1}{(x^2 + x + 1)^2} = \frac{2(1x^2 + 1x + 1)}{(x^2 + x + 1)^2} = \frac{2(x+1)^2}{(x+1)^2} = \frac{2}{x+1}$$

$$\rightarrow x+1 \neq 0 \quad x \neq -\frac{1}{x} \rightarrow D_f = \mathbb{R} - \left\{-\frac{1}{x}\right\}$$

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$$f(x) = \begin{cases} \frac{x(x^2-1) + 2(x^2-1)}{x^2-1} = \frac{(x^2-1)(x+2)}{(x^2-1)} & x \neq \pm 1 \rightarrow x+2 \\ \frac{ax+b}{x+c} & x = \pm 1 \end{cases}$$

$$\Rightarrow x+2 = \frac{ax+b}{x+c} \Rightarrow f(1) = 1+2 = 3 = \frac{a+b}{1+c} \rightarrow a+b = 3(1+c) \rightarrow a+b = 3+3c$$

$$\rightarrow b+c = 3+\frac{1}{x} = \frac{a}{x}$$

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$$g(x) = \frac{(x-1)(x+1)}{h(x) \cdot Q(x)} = f(x) = \frac{x+1}{x-c} \rightarrow \text{تساوی دینیماتور و صورت مساوی شود}$$

$$\rightarrow h(x) = x-1, \quad Q(x) = x-c$$

$$\rightarrow D_g = D_f \rightarrow D_g = \mathbb{R} - \{1, c\}, \quad D_f = \{c\} \Rightarrow c=1$$

$$\rightarrow x^2 + ax + b = (x-1)^2 = x^2 - 2x + 1 \rightarrow a = -2, \quad b = 1$$

$$\rightarrow a+b+c = -2+1+1 = 0$$

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$$g(x) = (x+1)^2(x-1) + (1-x)^2(1-x-1)$$

$$= (x+1)^2(x-1) + (-x-1)^2(-x) = (x+1)^2(x-1) + (-x-1)^2(-x)$$

$$\rightarrow (x^2+2x+1)(x-1) + (x^2+2x+1)(-x) = x^3-2x^2+x-1-x^3-2x^2-x-1 = -4x^2-2 = g(x)$$

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$1 - 2x = 1 \rightarrow 2x = -2 \rightarrow x = -1 \rightarrow x < 0 \Rightarrow f(x) = \sqrt[3]{-1} + 2 = 1$ $1 - 2x = -1 \rightarrow 2x = 2 \rightarrow x = 1 \rightarrow x > 0 \rightarrow f(-1) = 2\sqrt{1} = 2$ $\rightarrow f(-1) + f(2) = 1 + 2 = \underline{3}$	6
$y = x + \sqrt{x} \rightarrow x + \sqrt{x} - y = (\sqrt{x} + 2)(\sqrt{x} - 2) = 0 \rightarrow \sqrt{x} = -2 \rightarrow \text{no solution}$ $\sqrt{x} = 2 \rightarrow \text{no solution}$ $\rightarrow \sqrt{x} = 2 \Rightarrow x = 4 \rightarrow f(y) = (2)^2 + 2(2) = 2 \cdot 4 + 2 = \underline{10}$	7
$f(x-1) + f(1-x) = (x-1)^2 + (x-1) + (1-x)^2 + (1-x) = (x-1)^2 + (x-1) - (x-1)^2 - (x-1) = 0$	8
$D_g: b - 2x \geq 0 \rightarrow x \leq \frac{b}{2}, \quad D_f: 2x - 2a \geq 0 \rightarrow x \geq \frac{2a}{2}$ $\rightarrow D_g \cap D_f = I \rightarrow \frac{b}{2} = 1 \rightarrow b = 2, \quad \frac{2a}{2} = 1 \rightarrow a = \frac{2}{2}$ $2f(1) - g(1) = 2\sqrt{2(1)} - 2 + 2 - \sqrt{2 - 2(1)} - 2 = -2 - 2 = -4 \rightarrow 2k = -2 \rightarrow k = -1$ $\rightarrow 2a - \frac{b}{2} - k = 2 - 1 + 1 = \underline{2}$	9
$D_g: x \geq -1, \quad D_f: m - x \geq 0 \rightarrow x \leq m \quad D_{f \circ g} = D_f \cap D_g = [-1, m] = [-1, m]$ $\Rightarrow m = v$ $\rightarrow f(x) = \sqrt{x-1} + x \quad g(x) = \sqrt{x+1} \rightarrow f(2) - g(2) = 2 + 1 - \sqrt{3} = 3 - \sqrt{3}$ $\rightarrow 2 + u = 3 - \sqrt{3} \rightarrow u = 1 - \sqrt{3}$ $\rightarrow m + u = 1 - \sqrt{3} + v = \underline{1 - \sqrt{3} + v}$	10