

$$y = \sqrt{f(u) - u + 1} \quad f(u) = \begin{cases} u-1; [u] > r \rightarrow u > \Delta \\ u+r; [u] \leq r \rightarrow u < \Delta \end{cases}$$

(٢) ①

$$\left. \begin{array}{l} u > \Delta \rightarrow u-1-u+1 > 0 \rightarrow u > 0 \rightarrow (\Delta, +\infty) \\ u < \Delta \rightarrow u+r-u+1 > 0 \rightarrow r+u > 0 \rightarrow u > -\frac{r}{r} \rightarrow [-\frac{r}{r}, +\infty) \end{array} \right\} \xrightarrow{u} D_y = [-\frac{r}{r}, +\infty)$$

$$y = \frac{r^2 u^r + r^2 u^r + 1(u+r)}{(f(u) + f(u+1))^r} = \frac{r(1u^r + 1u^r + u+1)}{(f(u) + f(u+1))^r} = \frac{r(u+1)^r}{((u+1)^r)^r} = \frac{r(u+1)^r}{(u+1)^{r^2}} = \frac{r}{u+1}$$

$$\text{نکات: } f(u) + f(u+1) \neq 0 \quad (u+1)^r \neq 0 \Rightarrow u \neq -\frac{1}{r} \quad D_y = \mathbb{R} - \left\{ -\frac{1}{r} \right\}$$

$$f(u) = \begin{cases} \frac{u^r + u^r - u - r}{u^r - 1} & ; u \neq \pm 1 \\ \frac{au+r}{u+b} & ; u = \pm 1 \end{cases} \quad g(u) = u + c$$

(٢)

$$\frac{u^r + u^r - u - r}{u^r - 1} = \frac{(u-1)(u^r + ru + r)}{(u-1)(u+1)} = \frac{(u-1)(u+r)(u+1)}{(u-1)(u+1)} = u+r$$

$$\rightarrow u+r = u+c \Rightarrow \boxed{C=r}$$

$$u=1 \rightarrow r = \frac{a+r}{1+b} \Rightarrow r + rb = a+r$$

$$u=-1 \rightarrow 1 = \frac{-a+r}{-1+b} \Rightarrow \begin{cases} -1+b = -a+r \\ r+rb = r \Rightarrow \boxed{b = \frac{1}{r}} \end{cases}$$

$$b+c = \frac{1}{r} + r = \boxed{\frac{a}{r}}$$

$$f(u) = \frac{u+1}{u-c} \quad g(u) = \frac{u^r-1}{u^r+au+b} = \frac{(u-1)(u+1)}{(u-c)(u-1)} \rightarrow (u-1)^r = u^r - ru + 1$$

(٢) ②

$$D_f = \mathbb{R} - \{c\}$$

$$D_g = \mathbb{R} - \{1, c\} \rightarrow D_f = D_g \Rightarrow \boxed{C=1}$$

$$a = -r \quad b = 1$$

$$a+b+c = -r+1+1 = \boxed{0}$$

$$f(u) = (u-r)^r (u-1) \quad f(u+1) + f(1-u) = ?$$

$$f(u+1) = \left(\cancel{u-r}^{u-r}\right)^r (u+1-1) = \overbrace{(u^r - \cancel{ru^r} + ru^r)}^{u^r - ru^r + ru^r} = (u^r - ru^r + ru^r) \quad \left\{ \begin{array}{l} \oplus \\ \boxed{-1u^r} \end{array} \right.$$

$$f(1-u) = \left(\cancel{1-u-r}^{1-u-r}\right)^r (1-u-1) = \overbrace{(u^r + ru^r + ru^r)}^{-u^r - ru^r - ru^r} = (-u^r - ru^r - ru^r) \quad \text{②}$$

$$f(1-ru) = \begin{cases} \sqrt{ru} + r & u \leq 0 \\ r\sqrt{u} & u > 0 \end{cases} \quad f(-1) + f(r) = ?$$

$$f + 1 = \boxed{2} \quad \text{④}$$

$$1-ru = -1 \Rightarrow u = 1 \Rightarrow r\sqrt{1} = r \quad \checkmark$$

$$1-ru = r \Rightarrow u = -1 \Rightarrow \sqrt{-1} + r = 1 \quad \checkmark$$

$$f(u + \sqrt{u}) = u^r + ru \quad f(4) = ?$$

$$u + \sqrt{u} = 4 \xrightarrow{\sqrt{u}=t} t^2 + t - 4 = 0 \Rightarrow (t+r)(t-r) = 0 \quad \left\{ \begin{array}{l} t = -r \Rightarrow \sqrt{u} = -r \text{ (no)} \\ t = r \Rightarrow \sqrt{u} = r \Rightarrow u = r^2 \end{array} \right.$$

$$f(4) = r^r + r(r) = r^2 + 1r = \boxed{r^2 + 1r} \quad \checkmark \quad \text{⑤}$$

$$f(u) = u^r + u \quad f(u-1) + f(1-u) = ?$$

$$f(u-1) = (u-1)^r + (u-1)$$

$$f(1-u) = (1-u)^r + (1-u) = -(u-1)^r - (u-1)$$

$$\left\{ \begin{array}{l} \oplus \\ \boxed{0} \end{array} \right. \quad f(u-1) + f(1-u) = \cancel{(u-1)^r} + \cancel{(u-1)} - \cancel{(u-1)^r} - \cancel{(u-1)} = 0 \quad \text{①}$$

$$f(u) = \sqrt{ru - ra} + k - 1 \Rightarrow f(u) = \sqrt{ru - ra} + r + k \quad g(u) = \sqrt{b - ru} + rk$$

$$rf - g = \{(1, k)\} \rightarrow \frac{f}{g, f} = \{1\}$$

$$D_f = ru - ra \geq 0 \Rightarrow u \geq \frac{ra}{r} \quad D_g = b - ru \geq 0 \Rightarrow u \leq \frac{b}{r} \Rightarrow \frac{ra}{r} = \frac{b}{r} = 1 \Rightarrow \frac{b}{a} = \frac{r}{r} = 1 \Rightarrow b = a$$

$$r\sqrt{f - \left(\frac{r}{r}\right)r} + rk - r = \sqrt{f - f} - rk = k \Rightarrow rk = -r \Rightarrow k = -1 \quad \checkmark$$

$$ra - \frac{b}{r} - k = r\left(\frac{r}{r}\right) - \frac{r}{r} + 1 = r - r + 1 = 1 \quad \checkmark$$

$$f(u) = \sqrt{m-u} + n \quad g(u) = \sqrt{ru+r}$$

$$D_{f,g} = [-1, V] \quad f(u) \times g(u) = (\sqrt{m-u} + n)(\sqrt{ru+r})$$

$$u \geq -1 \quad m-u \geq 0 \Rightarrow u \leq m \quad D_{f,g} = [-1, V] \Rightarrow \boxed{m=V} \quad \checkmark$$

$$f(r) - g(r) = 4\sqrt{r} \Rightarrow \sqrt{V-r} + n - \sqrt{4+r} = 4\sqrt{r} \Rightarrow r+n-\sqrt{4+r} = 4\sqrt{r} \Rightarrow n = 4\sqrt{r} - r + \sqrt{4+r}$$

$$m+n = V + 4\sqrt{r} - r = \boxed{4\sqrt{r} + V} \quad \checkmark \quad \text{⑥}$$