

$$g = \sqrt{f(m) - m + 1} \quad m \geq 4$$

$$f(m) = \begin{cases} m-1 & ; m \geq 4 \\ m+3 & ; m \leq 3 \end{cases}$$

$$g = \begin{cases} \sqrt{m} & ; m \geq 4 \\ \sqrt{m+3} & ; m \leq 3 \end{cases}$$

$$g = \begin{cases} \sqrt{m-1-m+1} & ; m \geq 4 \\ \sqrt{f(m)+3-m+1} & ; m \leq 3 \end{cases}$$

$$\begin{aligned} &\rightarrow m \geq 0 \cup m \geq 4 \rightarrow m \geq 4 \\ &\rightarrow m+3 \geq 0 \rightarrow m \geq -3 \cup m \leq 3 \rightarrow -\frac{f}{p}, m \leq 3 \end{aligned} \quad \left\{ \begin{array}{l} y \in D = \left[-\frac{f}{p}, +\infty \right) \end{array} \right.$$

$$y = \frac{r^2 m^r + r^2 m^r + 11m + r}{(f(m) + f(m+1))^r} = \frac{r^2 (1m^r + 1m^r + 9m+1)}{(r(m+1)^r)^r} = \frac{r^2 (r(m+1)^r)}{(r(m+1)^r)^r} = \frac{r}{r(m+1)^r} = \frac{r}{r(m+1)^r}$$

$$D = \mathbb{R} - \left\{ -\frac{1}{r} \right\}$$

$$f(m) = \begin{cases} \frac{a^r + m^r - m - r}{m^r - 1} & ; m \neq \pm 1 \\ \frac{am + r}{m + b} & ; m = \pm 1 \end{cases}$$

$$g(m) = m + c$$

$$\frac{a^r + m^r - m - r}{m^r - 1} = \frac{m^r (m+r) - (m+r)}{m^r - 1} = \frac{(m+r)(m^r - 1)}{m^r - 1} = m+r \rightarrow m+r = m+c \rightarrow \boxed{c=r}$$

$$\begin{aligned} m=1 &\rightarrow \left. \begin{aligned} f(1) &= \frac{a+r}{1+b} \\ g(1) &= 1+r=r \end{aligned} \right\} \frac{a+r}{1+b} = r \rightarrow a+r = r+rb \rightarrow rb-a = -1 \quad \left\{ \begin{array}{l} rb=r \rightarrow \boxed{b=\frac{1}{r}} \\ \boxed{b+c=r, \Delta} \end{array} \right. \\ m=-1 &\rightarrow \left. \begin{aligned} f(-1) &= \frac{-a+r}{-1+b} \\ g(-1) &= -1+r=r \end{aligned} \right\} \frac{-a+r}{-1+b} = r \rightarrow r-a = b-1 \rightarrow a+b=r \end{aligned}$$

$$f(m) = \frac{m+1}{m-c} \quad g(m) = \frac{m^r-1}{m^r+am+b} = \frac{(m-1)(m+1)}{m^r+am+b}$$

چون تابع برابر اند، حاصل $(m-1)$ از صورت $g(m)$ با همین عامل در مخرج زده شده همین چون تنها یک عدد در دانه $f(m)$ نیست پس x در دانه y هم فقط یک عدد باید باشد در نتیجه به صورت $(m-1)^r$ بهر دو باقی بماند m^r ، a و b باید یک است.

$$(m-1)^r = m^r - r m + 1 = m^r + a m + b \rightarrow \boxed{a=-r} \quad \boxed{b=1}$$

$$m-c = m-1 \rightarrow \boxed{c=1} \rightarrow \boxed{a+b+c=0}$$

$$f(m) = (m-r)^r (m-1)$$

$$f(m+1) = (m+1-r)^r (m+1-1) = (m-r)^r (m)$$

$$f(1-m) = (1-m-r)^r (m+1-1) = -(r-a)^r (m) = -(-m-r)^r (m)$$

$$g(m) = f(m+1) + f(1-m) = (m-r)^r (m) + (- (m+r)^r (m)) = m ((m-r)^r - (m+r)^r) = \boxed{-m^r}$$

$$f(1-m) = \begin{cases} \sqrt{m} + r & ; m \leq 0 \\ \sqrt{m} & ; m > 0 \end{cases}$$

$$1-rm = -1 \rightarrow m=1 \rightarrow f(-1) = r$$

$$1-rm = +r \rightarrow m=-1 \rightarrow f(r) = -1+r=1$$

$$f(-1) + f(r) = r \rightarrow \boxed{f(-1) + f(r) = \Delta}$$

$$f(n+\sqrt{n}) = n^{\frac{1}{2}} + \sqrt{n}$$

$$n+\sqrt{n} = 4 \Rightarrow n+\sqrt{n}-4=0 \Rightarrow (\sqrt{n}-2)(\sqrt{n}+2)=0 \Rightarrow \sqrt{n}=2 \Rightarrow n=4$$

$$\Rightarrow f(4) = 4^{\frac{1}{2}} + \sqrt{4} = 2\sqrt{4} + 2 = \boxed{2\sqrt{4}}$$

$$f(n) = n^{\frac{1}{2}} + n$$

$$f(n-1) = (n-1)^{\frac{1}{2}} + (n-1)$$

$$f(1-n) = (1-n)^{\frac{1}{2}} + (1-n) = -(n-1)^{\frac{1}{2}} - (n-1)$$

$$\left\{ \begin{array}{l} f(n-1) + f(1-n) = 0 \end{array} \right.$$

$$f(n) = \sqrt{fn - \frac{1}{f}a} + k-1 \xrightarrow{\text{مثلا}} fn - \frac{1}{f}a \geq 0 \Rightarrow n \geq \frac{1}{f}a$$

$$g(n) = \sqrt{b - fn} + \frac{1}{f}k \xrightarrow{\text{مثلا}} b - fn \geq 0 \Rightarrow n \leq \frac{b}{f}$$

$$f \cdot g = \left\{ (1, k) \right\} \rightarrow \text{با توجه به اینکه } f \text{ و } g \text{ دو تابع دایره ای هستند و } f \cdot g = 1 \text{ پس } f = \frac{1}{g} \text{ و } g = \frac{1}{f} \rightarrow \frac{1}{f}a = 1 \rightarrow \boxed{a = \frac{f}{1}} \\ \text{همچنین باید عدد صحیحی برای } k \text{ پیدا کنیم.} \quad \frac{b}{f} = 1 \rightarrow \boxed{b = f}$$

$$\left. \begin{array}{l} f(1) = \sqrt{f - \frac{1}{f}} + k-1 = k-1 \\ g(1) = \sqrt{f - f} + \frac{1}{f}k = \frac{1}{f}k \end{array} \right\} f \cdot g(1) = \frac{1}{f}k - \frac{1}{f}k = k \rightarrow \boxed{k = -1}$$

$$\frac{1}{f}a - \frac{b}{f} - k = \frac{1}{f}\left(\frac{f}{1}\right) - \frac{f}{f} - (-1) = \boxed{1}$$

$$f(n) = \sqrt{m-n} + n \rightarrow D = (-\infty, \sqrt{m}]$$

$$g(n) = \sqrt{2n+1} \rightarrow D = [-1, +\infty)$$

$$D_{f \cdot g} = \underbrace{D_f}_{(-\infty, \sqrt{m}]} \cap \underbrace{D_g}_{[-1, +\infty)} = [-1, \sqrt{m}] \rightarrow \boxed{m = 1}$$

$$\left. \begin{array}{l} f(x) = \sqrt{1-x} + x = 1+x \\ g(x) = \sqrt{2x+1} = \sqrt{1} = 1 \end{array} \right\} f \cdot g(x) = 1+x - 1\sqrt{1} = x \rightarrow \boxed{x = 1\sqrt{1} - 1}$$

$$\boxed{m+n = 1+1\sqrt{1} - 1 = 1+1\sqrt{1}}$$