

به نام خدا زهرای زارینا

$$(1) f(n) \begin{cases} n-1 & n \geq 0 \\ n+1 & n < 0 \end{cases}$$

$$y \begin{cases} x \geq 0 \rightarrow \sqrt{x} \geq 0 \\ x < 0 \rightarrow \sqrt{x+1} \geq 0 \rightarrow -\frac{1}{4} \leq x \end{cases} \Rightarrow D = \left[-\frac{1}{4}, +\infty\right)$$

$$(2) \frac{r(n^r + 12n^r + 9n + 1)}{(r+1)^r} = \frac{r(r+1)}{(r+1)^r} \rightarrow \frac{-\frac{1}{r}}{-\frac{1}{r} + 0} \rightarrow D = \mathbb{R} \setminus \left\{-\frac{1}{r}\right\}$$

$$(3) f(n) \begin{cases} \frac{n^r + rn - n - r}{(n-1)(n+1)} & , n \neq \pm 1 \\ \frac{an+r}{n+b} & , n = \pm 1 \end{cases}$$

$$x=0 \rightarrow r = 0 + C \rightarrow C = r$$

$$\begin{aligned} x=1 &\rightarrow \frac{a+r}{1+b} = r \rightarrow a+r = r + rb \\ x=-1 &\rightarrow \frac{r-a}{b-1} = 1 \rightarrow r-a = b-1 \end{aligned} \left\{ \begin{aligned} f &= r + fb \rightarrow b = \frac{1}{r} \\ a &= \frac{a}{r} \end{aligned} \right.$$

$$a+b+C = r + \frac{1}{r} + \frac{a}{r} = \boxed{a}$$

$$(4) \left. \begin{aligned} f(n) &= \frac{n+1}{n-1} \rightarrow \frac{n+1}{n-1} \\ g(n) &= \frac{n^r-1}{n^r+an+b} \rightarrow \frac{(n+1)(n-1)}{(n-1)^r} \end{aligned} \right\} \begin{aligned} a &= -1 \\ b &= +1 \\ C &= +1 \end{aligned} \Rightarrow a+b+C = \boxed{0}$$

$$(5) (n+1-r)^r (n+1-1) + (1-n-r)^r (1-n-1)$$

$$(n-r)^r(n) + (-n-r)^r(-n) = n^r + rn - rn^r - n^r - rn - rn^r = \boxed{-2n^r}$$

$$(9) \quad f(1-x) \begin{cases} \sqrt[3]{x} + 1 & x \leq 0 \\ x\sqrt{x} & x > 0 \end{cases}$$

$$\left. \begin{aligned} 1-x &= 1 \rightarrow x = 0 \\ 1-x &= -1 \rightarrow x = 2 \end{aligned} \right\} f(1) + f(-1) = \sqrt[3]{-1} + 1 + 2\sqrt{2} = \boxed{5}$$

$$(10) \quad x + \sqrt{x} = 9 \rightarrow x + \sqrt{x} - 9 \rightarrow (\sqrt{x} + 1)(\sqrt{x} - 2) = 0 \rightarrow x = 4$$

$$f(4) = 4^{\sqrt{4}} + 1\sqrt{4} = 16 \cdot 4 + 2 = \boxed{66}$$

$$(11) \quad \left. \begin{aligned} f(x-1) &= (x-1)^{\sqrt{x}} + (x-1) \\ f(1-x) &= -(x-1)^{\sqrt{x}} - (x-1) \end{aligned} \right\} f(x-1) + f(1-x) = \boxed{0}$$

$$(12) \quad \left. \begin{aligned} f(1) &= \sqrt{f+ka} + k+1 \\ g(1) &= \sqrt{b-f} + rk \end{aligned} \right\} \begin{aligned} f \cdot f - g(1) &= \sqrt{f+ka} + k+1 - \sqrt{b-f} - rk \\ rk - r^2k - r &= k \rightarrow rk = -r \rightarrow k = -1 \end{aligned}$$

$$\rightarrow ka < f \rightarrow a \leq \frac{f}{r} \quad , \quad b \geq f \rightarrow ra \leq \frac{b}{r} - k = f - r + 1 = \boxed{1}$$

$$(13) \quad \left. \begin{aligned} g &= \sqrt{m+r} \rightarrow D = [-1, +\infty) \\ f &= \sqrt{m-n} + n \rightarrow D = (-\infty, m] \end{aligned} \right\} D = [-1, 5] \rightarrow m = 5$$

$$f - g(1) = \sqrt{f} + n - \sqrt{n} = 1 - 2\sqrt{1} + n = 9\sqrt{1} \rightarrow n = 11\sqrt{1} - 1$$

$$m+n = \boxed{11\sqrt{1} + 5}$$