

۱۲، ۵

مجموعه های باز و بسته

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نکته: دایره ها و مربع ها

۱) $f(x) = x+1$ $\rightarrow$ $[x] > f \rightarrow f(x) = x+1$

۲) $f(x) = \begin{cases} x-1 & , [x] > f \\ f(x+1) & , [x] \leq f \end{cases}$ $\rightarrow$ $y = \sqrt{f(x) - x + 1}$

صفحه آخر...

$$\frac{x^3(1+x^2+x^4+x^6+x^8+x^{10}+x^{12}+x^{14}+x^{16}+x^{18}+x^{20}+x^{22}+x^{24}+x^{26}+x^{28}+x^{30}+x^{32}+x^{34}+x^{36}+x^{38}+x^{40}+x^{42}+x^{44}+x^{46}+x^{48}+x^{50}+x^{52}+x^{54}+x^{56}+x^{58}+x^{60}+x^{62}+x^{64}+x^{66}+x^{68}+x^{70}+x^{72}+x^{74}+x^{76}+x^{78}+x^{80}+x^{82}+x^{84}+x^{86}+x^{88}+x^{90}+x^{92}+x^{94}+x^{96}+x^{98}+x^{100})}{(x+1)^{101}} \rightarrow \frac{x^3(1+x^2+x^4+x^6+x^8+x^{10}+x^{12}+x^{14}+x^{16}+x^{18}+x^{20}+x^{22}+x^{24}+x^{26}+x^{28}+x^{30}+x^{32}+x^{34}+x^{36}+x^{38}+x^{40}+x^{42}+x^{44}+x^{46}+x^{48}+x^{50}+x^{52}+x^{54}+x^{56}+x^{58}+x^{60}+x^{62}+x^{64}+x^{66}+x^{68}+x^{70}+x^{72}+x^{74}+x^{76}+x^{78}+x^{80}+x^{82}+x^{84}+x^{86}+x^{88}+x^{90}+x^{92}+x^{94}+x^{96}+x^{98}+x^{100})}{(x+1)^{101}} = \frac{x^3}{x+1}$$

$f(x) = (x+1) \times x^3 = x^3 + 3x^2 + 3x + 1$

$Df = \mathbb{R} - \{-1\}$

$$\frac{x^3+x^2-x-1}{x^2-1} \rightarrow \frac{(x-1)(x+1)(x+2)}{(x-1)(x+1)} \rightarrow x+2$$

$g(x) = x+c$, $f(x) = \begin{cases} \frac{x^3+x^2-x-1}{x^2-1} & , x \neq \pm 1 \\ \frac{ax^2+bx+c}{x^2-1} & , x = \pm 1 \end{cases}$

صفحه آخر...

$g(x) \Rightarrow \frac{(x-1)(x+1)}{x^2+ax+b}$

$b=1$, $a=-1$

$g(x) = \frac{(x-1)(x+1)}{(x-1)^2} = \frac{x+1}{x-1} = f(x) = \frac{x+1}{x-c}$

$-1+1+1 = 1$

$g(x) = f(x+1) + f(1-x)$, $f(x) = (x-1)^2(x-1)$

$f(1-x) \Rightarrow (1-x-1)^2(1-x-1) \Rightarrow f(1-x) = (-x)^2(-x) = -x^3$

$f(x+1) \Rightarrow (x+1-1)^2(x+1-1) \Rightarrow f(x+1) = x^2 \cdot x = x^3$

$g(x) \Rightarrow -x^3 + x^3 = 0$

1) $1 - \sqrt{x} = 1^0 \rightarrow \sqrt{x} = -1 \rightarrow x = -1$ $\Rightarrow \sqrt{-1} + 1 \Rightarrow -1 + 1 = 0$ ✓

2) $1 - \sqrt{x} = -1 \rightarrow \sqrt{x} = 2 \rightarrow x = 4$ ✓

$1 + \sqrt{x} = 5$ ✓

جواب

$x + \sqrt{x} = 4 \xrightarrow{\sqrt{x}=t} t^2 + t = 4 \Rightarrow t^2 + t - 4 = 0$

$(t+4)(t-3) = 0$
 $\textcircled{-3} \quad \textcircled{4}$

$f(x) = x^2 + 1$ $f(x + \sqrt{x}) = x^2 + 1 + 2x\sqrt{x}$
 $\sqrt{x} = 2 \Rightarrow x = 4$ ✓
 $\sqrt{x} = -3 \Rightarrow x = 9$

$f(4 + \sqrt{4}) = (4 + 2)^2 + 1 = 36 + 1 = 37$ ✓
 $(4 + \sqrt{4})^2 = 36$

$f(x-1) + f(1-x)$

$f(x) = x^2 + 1$ $f(x-1) = (x-1)^2 + 1 = x^2 - 2x + 2$

1) $f(x-1) \Rightarrow (x-1)((x-1)^2 + 1) \Rightarrow (x-1)(x^2 - 2x + 2)$

2) $f(1-x) \Rightarrow (1-x)((1-x)^2 + 1) \Rightarrow (1-x)(1 - 2x + x^2 + 1) \Rightarrow (1-x)(x^2 - 2x + 2)$

$\Rightarrow \textcircled{1} + \textcircled{2} \Rightarrow x^2 - 2x + 2(x-1 + 1-x) \Rightarrow 0$ ✓

اگر $f(u) = \sqrt{u - a} + k - 1$ و $g(u) = \sqrt{b - u} + k$ و $f - g = \{(1, k)\}$ معادله $\frac{b}{f} - k$ معادله $f - g = \{(1, k)\}$ معادله $\frac{b}{f} - k$ معادله $f - g = \{(1, k)\}$

$$g(u) \xrightarrow{\text{if } u=1} \sqrt{b - 1} + k \quad \sqrt{f - a} + k - 1 -$$

$$f(u) \xrightarrow{\text{if } u=1} \sqrt{f - a} + k - 1$$

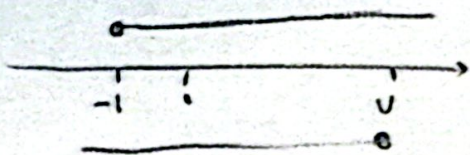
صفحه آفر...

برای تابع $f(u) = \sqrt{u - a} + n$ و $g(u) = \sqrt{u + 2}$ اگر $D_{fg} = [-1, 7]$ و $f - g = c\sqrt{f}$ معادله $m + n$ معادله $f - g = c\sqrt{f}$ معادله $m + n$ معادله $f - g = c\sqrt{f}$

$$g(u) = \sqrt{u + 2} = \sqrt{u} = \sqrt{u} \checkmark$$

$$f = c\sqrt{f} + \sqrt{f} \Rightarrow f = E\sqrt{f} \Rightarrow f(u) = \sqrt{f} \Rightarrow \sqrt{m - u} + n = E\sqrt{f}$$

$$u + n = E\sqrt{f} \rightarrow n = E\sqrt{f} - u \rightarrow n = 2(\sqrt{f} - 1)$$



$m = 7$ طبق گفته باید u باشد که اشتباه است با دامنه تابع f باشد

$$[-1, 7]$$

$$u + 2(\sqrt{f} - 1)$$

صفحه آفر...

$$f(n) = \begin{cases} r_{n-1} & n \geq \omega \\ r_{n+r} & n < \omega \end{cases}$$

-1

$$\begin{cases} n \geq \omega \rightarrow y = \sqrt{r_{n-1} - n + 1} \rightarrow y = \sqrt{n} & n \geq 0 \xrightarrow{n} n \geq \omega \\ n < \omega \rightarrow y = \sqrt{r_{n+r} - n + 1} \rightarrow y = \sqrt{r_{n+r}} & n \geq -\frac{r}{r} \xrightarrow{n} -\frac{r}{r} \leq n \leq \omega \end{cases}$$

$$D_y = \left[-\frac{r}{r}, +\infty\right)$$

-2

$$y = \frac{r(rn^r + 1rn^r + 4n + 1)}{(rn + 1)^r} = \frac{r(rn + 1)^r}{(rn + 1)^r} = \frac{r}{rn + 1}$$

$$\Rightarrow D_y = \left[R - \frac{1}{r}\right]$$

(2) ضابطه های مساوی

3- برای تساوی روابط (1) دامنه های برابر

$$f(n) = \begin{cases} \frac{(n-1)(n^r + rn + r)}{(n-1)(n+1)} = n + r \\ \frac{an + r}{n + b} \end{cases}$$

$$n \neq \mp 1 \Rightarrow C = r$$

$$n = \mp 1$$

مقدار تابع در $n = \mp 1$ باید در روابط f و g برابر باشند:

$$\left\{ \begin{array}{l} \rightarrow a = r, \omega, b = 0, \omega \rightarrow b + C = r, \omega \end{array} \right.$$

$$f(1) = \frac{a+r}{1+b} = r \rightarrow a - rb = 1$$

$$f(-1) = \frac{-a+r}{-1+b} = 1 \rightarrow a + b = r$$

9. اشتراك دالة f و g فقط $a=1$ استيعا :

$$D_f = \left[\frac{r_a}{r}, +\infty \right) \quad D_g = (-\infty, \frac{b}{r}] \xrightarrow{\cap} \{1\}$$

$$\frac{r_a}{r} = \frac{b}{r} = 1 \rightarrow a = \frac{r}{r} \quad b = r$$

$$r f(1) - g(1) = K \rightarrow r K - r - r K = K \rightarrow K = -1$$

$$r a - \frac{b}{r} = K = \textcircled{r}$$

$$D_f = (-\infty, m]$$

$$\Rightarrow D_{f \cap g} = \{-1, v\} \rightarrow \boxed{m = v}$$

$$D_g = [-1, +\infty)$$

$$(f-g)(r) = r + n - r\sqrt{r} = 4\sqrt{r} \rightarrow \boxed{n = 1\sqrt{r} - r}$$

$$m + n = \boxed{1\sqrt{r} + \Delta}$$

-1.