

$$P_n = \begin{cases} r_{n-1} & [n] > r \Rightarrow n \geq a \Rightarrow r_{n-1} - n + 1 \geq 0 \Rightarrow n \geq 0 \Rightarrow \begin{cases} n \geq 0 \\ n \geq a \end{cases} \\ r_{n+r} & [n] < r \Rightarrow n < a \Rightarrow r_{n+r} - n + 1 \geq 0 \Rightarrow r_{n+r} \geq 0 \Rightarrow n \geq -\frac{r}{r} \Rightarrow \begin{cases} n < a \\ n \geq -\frac{r}{r} \end{cases} \Rightarrow -\frac{r}{r} \leq n < a \end{cases}$$

$$D_y = ?$$

$$y = \sqrt{r(n) - n + 1}$$

$$n \geq a \cup -\frac{r}{r} \leq n < a \Rightarrow D_y = \left[-\frac{r}{r}, +\infty \right)$$

$$y = \frac{r_{n+r} + r_{n+r} + 1}{(r_{n+r} + r_{n+r})^r} \Rightarrow \frac{r(1 + r_{n+r} + r_{n+r} + 1)}{(1 + r_{n+r} + r_{n+r})^r} = \frac{r(r_{n+r} + 1)^r}{(r_{n+r} + 1)^r} = \frac{r}{r_{n+r} + 1}$$

$$D_y = ?$$

$$\frac{r}{r_{n+r} + 1} \Rightarrow r_{n+r} + 1 \neq 0$$

$$D_y = \{R - \frac{1}{r}\}$$

$$r_{n+r} + 1$$

$$n \neq -\frac{1}{r}$$

$$P_n = \begin{cases} \frac{r_{n+r} + r_{n+r} - n - r}{n^{r-1}}, & n \neq \pm 1 \\ \frac{a_{n+r}}{n+b}, & n = \pm 1 \end{cases} \quad \frac{(n-1)(n+r+1)}{(n-1)(n+1)} \Rightarrow \frac{(n+1)(n+r)}{(n+1)} = n+r$$

$$n+C = n+r \Rightarrow C = r$$

$$b+C = r + \frac{1}{r} = \frac{a}{r}$$

$$\begin{aligned} g(n) = n+C & \quad \text{if } n=1 \rightarrow \frac{a+r}{1+b} = r \rightarrow a+r = a+r \\ b+C = ? & \quad \text{if } n=-1 \rightarrow \frac{-a+r}{-1+b} = 1 \rightarrow -1+b = -a+r \end{aligned} \quad \left. \begin{aligned} -rb = -r \\ b = \frac{1}{r} \end{aligned} \right\}$$

$$P_n = \frac{n+1}{n-C} \Rightarrow P_n = g(n) \quad a+b+C = -r+1+1 = 0$$

$$g(n) = \frac{n^r-1}{ar+an+b} \quad \frac{(n-1)(n+1)}{(n-1)(n+1)} = \frac{n+1}{n-C} \Rightarrow C = +1$$

$$a+b+C = ?$$

$$n^r + a + b \rightarrow (n-1)^r$$

$$n^r + 1 - r_n \rightarrow a = -r, b = 1$$

$$P_{n+1} = (n-r)^r (n-1)$$

$$\Rightarrow n^r + r_{n+1} - r_{n+1} - n^r - r_n - r_{n+1} = -\frac{1}{r} n^r$$

$$g_{n+1} = P_{n+1} + P_{n+1}$$

$$P_{n+1} = (n+1-r)^r (n+1-1)^r = n^r + r_{n+1} - r_{n+1}$$

$$P_{n+1} = (1-n-r)^r (1-n-1) = -n^r - r_{n+1} - r_{n+1}$$

$$f(1-r_n) = \begin{cases} \sqrt{n} + r & n \leq 0 \\ r\sqrt{n} & n > 0 \end{cases}$$

$$f(-1) + f(r) = ? \quad \leadsto \quad r+1 = \underline{a}$$

$$f(-1) \Rightarrow 1-r_n = -1 \rightarrow n = 1 \Rightarrow r\sqrt{1} = 2$$

$$f(r) \Rightarrow 1-r_n = r \rightarrow n = -1 \Rightarrow r\sqrt{-1} + r = 1$$

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$$f(n+\sqrt{n}) = n^r + r_n$$

$$f(4) = ? \quad r^r + r \cdot r = r \cdot 4 + 1r = \underline{r \cdot 4 + 1r}$$

$$n + \sqrt{n} = 4 \Rightarrow \sqrt{n} = 4 - n \Rightarrow n = r \cdot 4 + n^r - 1r_n$$

$$n^r - 1r_n + r \cdot 4 = 0 \quad (n-r)(n-4) = 0$$

$$\text{if } n = r \Rightarrow r + \sqrt{r} = 4 \quad \checkmark$$

$$\text{if } n = 4 \Rightarrow 4 + \sqrt{4} = 1r \quad \times$$

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$$f(n) = n^r + n$$

$$f(n-1) + f(1-n) = ?$$

$$\Rightarrow (n-1)^r + n-1 = n^r + r_n - r_n^r - 1 - n-1$$

$$\Rightarrow (-n+1)^r + 1-n = 1 - n^r - r_n + r_n^r + 1 - n$$

$$n^r + r_n - r_n^r - 1 - n - 1 + 1 - n^r - r_n + r_n^r + 1 - n \Rightarrow \underline{0}$$

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$$f_n = \sqrt{r_n - r_a} + k - 1 \quad r_n - r_a \geq 0$$

$$-r_a \geq -r_n \Rightarrow n \geq \frac{r_a}{r} \quad n=1 \Rightarrow \frac{r_a}{r} = 1 \Rightarrow a = \frac{r}{2}$$

$$g_n = \sqrt{b - r_n} + r_k \quad b - r_n \geq 0 \rightarrow n \leq \frac{b}{r} \quad n=1 \Rightarrow \frac{b}{r} = 1 \Rightarrow b = r$$

$$f \cdot g = f(1, k) \quad f(1) - g(1) = r_k - r - r_k = -k - r$$

$$k = -1$$

$$r_a - \frac{b}{r} - k = ?$$

$$r_a - \frac{b}{r} - k = r \cdot \frac{r}{r} - r - (-1) = \underline{r}$$

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$$f(n) = \sqrt{m-n} + n = m-n \geq 0 \quad n \leq m \quad m \in D_f \cap [1, \infty) \quad m = v$$

$$g(n) = \sqrt{r_n + r} \Rightarrow r_n + r \geq 0 \quad n \geq -1$$

$$D_{f \cdot g} = [-1, v]$$

$$f \cdot g(r) = 9\sqrt{r}$$

$$m+n = ?$$

$$f(r) - g(r) = \sqrt{v-r} + n - \sqrt{4+r} = r+n - r\sqrt{r} \pm 4\sqrt{r}$$

$$n = \sqrt{v-r} - r$$

$$m+n = \sqrt{v-r} - r + v = \underline{\sqrt{v-r} + v}$$

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