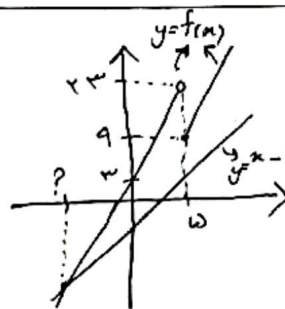


۲۰ آفرینش

$$f(x) = \begin{cases} 2x-1 & x \geq 5 \\ 2x+3 & x < 5 \end{cases} \quad f(x) - x + 1 \geq 0 \Rightarrow f(x) \geq x-1$$

$$2x+3 = x-1 \Rightarrow x = -\frac{4}{1} \text{ فقط } (x < 5)$$

$$\Rightarrow D = \left[-\frac{4}{1}, +\infty\right)$$



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$$\frac{3(1x^3 + 12x^2 + 4x + 1)}{(2x+1)^4} = \frac{3(2x+1)^3}{(2x+1)^4} = \frac{3}{2x+1} \rightarrow x \neq -\frac{1}{2}$$

$$D = \mathbb{R} \setminus \left\{-\frac{1}{2}\right\}$$

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$$x \neq \pm 1 \Rightarrow \frac{x^3 - x + 2x^2 - 2}{x^2 - 1} = x + c \Rightarrow \frac{x(x^2 - 1) + 2(x^2 - 1)}{x^2 - 1} = x + 2 = x + c \Rightarrow c = 2$$

$$x = 1 \Rightarrow \frac{a+2}{1+b} = 1+2 \Rightarrow 3+b = a+2 \Rightarrow \begin{cases} -a+3b = -1 \\ a+b = 3 \end{cases}$$

$$x = -1 \Rightarrow \frac{-a+2}{-1+b} = -1+2 \Rightarrow b-1 = 2-a \Rightarrow \begin{cases} -a+3b = -1 \\ a+b = 3 \end{cases} \Rightarrow b = 2 \Rightarrow a = \frac{5}{2}$$

$$b+c = \frac{1}{2} + 2 = \frac{5}{2}$$

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$$\frac{x^2-1}{x^2+ax+b} = \frac{x+1}{x-c} \Rightarrow \frac{x^2-1}{x^2+ax+b} = \frac{(x+1)(x-1)}{(x-c)(x-1)} \Rightarrow \frac{x^2-1}{x^2+ax+b} = \frac{x^2-1}{x^2-(c+1)x+c}$$

$$\Rightarrow \begin{cases} a = -1-c \\ b = c \end{cases} \quad D_g = D_f \Rightarrow D_g = \mathbb{R} - \{1\} \Rightarrow x-c = 1 \Rightarrow c = x-1 \Rightarrow c = 1$$

$$a+b+c = -1-1+2 = 0$$

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$$f(x+1) = (x+1-3)^2(x+1-1)^2 = x(x-2)^2$$

$$f(1-x) = (1-x-3)^2(1-x-1) = -x(x+2)^2$$

$$g(x) = x(x-2)^2 - x(x+2)^2 = x((x-2)^2 - (x+2)^2)$$

$$= x(x^2 - 4x + 4 - x^2 - 4x - 4) = -8x^2$$

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$$1 - 2x = -1 \Rightarrow x = 1 \Rightarrow f(-1) = f(1-2(1)) = \sqrt{1} = 1$$

$$1 - 2x = 3 \Rightarrow x = -1 \Rightarrow f(3) = f(1-2(-1)) = \sqrt{1} + 2 = -1 + 2 = 1$$

$$f(-1) + f(3) = 1 + 1 = \boxed{2}$$

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$$x + \sqrt{x} = 9 \Rightarrow x = 16 \Rightarrow f(9) = f(9 + \sqrt{9}) = 9^2 + 3 \times 9 = 9 \Delta 9 + 12 = \boxed{96}$$

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$$f(x-1) = (x-1)^2 + (x-1) \quad f(1-x) = (1-x)^2 + (1-x) = (-(x-1))^2 + (-(x-1)) = -(x-1)^2 - (x-1)$$

$$f(x-1) + f(1-x) = (x-1)^2 + (x-1) - (x-1)^2 - (x-1) = \boxed{0}$$

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$$D_{f \cap g} = \{1\} \Rightarrow D_g \cap D_f = \{1\} \quad D_f: (x-3a) > 0 \Rightarrow x > \frac{3a}{2} \Rightarrow \frac{b}{2} = \frac{3a}{2} = 1$$

$$\Rightarrow \begin{cases} a = \frac{2}{3} \\ b = 1 \end{cases} \Rightarrow \begin{cases} f(x) = \sqrt{x-1} + K - 1 \\ g(x) = \sqrt{1-x} + 3K \end{cases} \quad \forall f(1) - g(1) = K \Rightarrow$$

$$2(K-1) - 3K = K \Rightarrow 2K - 2 = K \Rightarrow K = -2 \Rightarrow K = -1$$

$$3a - \frac{b}{2} - K = 3 \times \frac{2}{3} - \frac{1}{2} - (-1) = 2 - \frac{1}{2} + 1 = \boxed{\frac{5}{2}}$$

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$$D_{f \cap g} = [-1, V] \Rightarrow D_f \cap D_g = [-1, V] \quad D_g: (x+2) > 0 \Rightarrow x > -2 \Rightarrow m = V$$

$$f-g(x) = \sqrt{V-3} + n - \sqrt{2(3)+2} = 4\sqrt{2}$$

$$2+n - 2\sqrt{2} = 4\sqrt{2} \Rightarrow n = 6\sqrt{2} - 2$$

$$D_f: (m-x) > 0 \Rightarrow x < m$$

$$\Rightarrow m+n = V + 6\sqrt{2} - 2 = \boxed{6\sqrt{2} + \Delta}$$

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