

$$f(x) = \begin{cases} \sqrt{x-1} & ; x \geq 1 \\ \sqrt{x+1} & ; x \leq -1 \end{cases} \Rightarrow f(x) = \begin{cases} \sqrt{x-1} & ; x \geq 1 \\ \sqrt{x+1} & ; x \leq -1 \end{cases} \quad (1)$$

$$y = \sqrt{f(x) - x + 1} \Rightarrow f(x) - x + 1 \geq 0 \quad \begin{cases} x \geq 1 \Rightarrow \sqrt{x-1} - x + 1 \geq 0 \Rightarrow x \geq 0 \quad (2) \\ x \leq -1 \Rightarrow \sqrt{x+1} - x + 1 \geq 0 \Rightarrow x \geq -\frac{1}{2} \quad (3) \end{cases}$$

$$\begin{cases} (1) \cap (2) \Rightarrow x \geq 1 \quad (I) \\ (2) \cap (3) \Rightarrow -\frac{1}{2} \leq x < 1 \quad (II) \end{cases} \Rightarrow I \cup II \Rightarrow D_f = \left[-\frac{1}{2}, +\infty\right)$$

$$y = \frac{\sqrt{x-1} + \sqrt{x+1} + 1}{(\sqrt{x-1} + \sqrt{x+1})^2} = \frac{\sqrt{x-1} + \sqrt{x+1} + 1}{(2x+1)^2} = \frac{\sqrt{x-1} + \sqrt{x+1} + 1}{(2x+1)^2} \quad (2)$$

$$\Rightarrow \frac{1}{2x+1} \quad D_f \Rightarrow 2x+1 \neq 0 \Rightarrow x \neq -\frac{1}{2} \Rightarrow D_f = \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

$$D_f = (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, +\infty)$$

$$f(x) = \begin{cases} \frac{(x+1)(x^2-1)}{(x^2-1)} & ; x \neq \pm 1 \\ \frac{ax+b}{x+c} & ; x = \pm 1 \end{cases} \Rightarrow f(x) = \begin{cases} x+1 & ; x \neq \pm 1 \\ \frac{ax+b}{x+c} & ; x = \pm 1 \end{cases} \quad (3)$$

$$g(x) = x+1$$

چون $g(x) = f(x)$ بی $x=1$

$$x=1 \rightarrow 1 = \frac{a+1}{1+b} \Rightarrow a+1 = 1+b \Rightarrow a = b$$

$$x=-1 \rightarrow \frac{-a+b}{-1+b} = 1 \Rightarrow a = b-1$$

$$1+b = b-1 \Rightarrow b = -1$$

$$b+c = -1+c = -1 \Rightarrow c = 0$$

$$g(x) = f(x) \Rightarrow \frac{(x-1)(x+1)}{(x^2+ax+b)} = \frac{(x+1)}{(x-c)} \quad (4)$$

$$x^2+ax+b = (x-1)(x-c) \Rightarrow x^2 - (c+1)x + c \Rightarrow \begin{cases} b = c \\ a = -(c+1) \end{cases}$$

$$x-1 = x-c \Rightarrow c=1 \quad b=1 \quad a=-2 \rightarrow a+b+c = 0$$

$$f(x) = (x-1)^2(x-1)$$

$$f(x+1) = (x+1-1)^2(x+1-1) = (x-1)^2(x) \quad (5)$$

$$f(1-x) = (1-x-1)^2(1-x-1) = (-x-1)^2(-x) = -x(x+1)^2$$

$$g(x) = f(1+x) + f(1-x) = (x-1)^2x - x(x+1)^2$$

$$\Rightarrow x[(x^2-2x+1) - (x^2+2x+1)] = x(-4x) = -4x^2$$

$$f(1-\sqrt{x}) = \begin{cases} \sqrt{x} + 1 & x \leq 0 \\ \sqrt{x} & x > 0 \end{cases} \Rightarrow \boxed{f(-1) + f(1) = 2} \quad (2) \textcircled{6}$$

$$f(-1) \Rightarrow 1 - \sqrt{x} = -1 \Rightarrow \boxed{x = 1} \xrightarrow{x > 0} \sqrt{1} = 1 \quad \checkmark$$

$$f(1) \Rightarrow 1 - \sqrt{x} = 1 \Rightarrow \boxed{x = 0} \xrightarrow{x \leq 0} \sqrt{-1} + 1 = 1 \quad \checkmark$$

$$f(x + \sqrt{x}) = x^2 + \sqrt{x} \Rightarrow (1)^2 + \sqrt{1} = 1 + 1 = 2 \quad \checkmark \quad (2) \textcircled{7}$$

$$f(1) \Rightarrow x + \sqrt{x} = 1 \Rightarrow \boxed{x = 1} \quad \checkmark$$

$$f(x-1) + f(1-x) = ?$$

$$f(x) = x^2 + x$$

$$x-1 = t \Rightarrow \boxed{x = t+1} \Rightarrow f(x-1) = (t+1)^2 + t+1$$

$$1-x = t \Rightarrow \boxed{x = -t+1} \Rightarrow f(1-x) = (1-x)^2 + (1-x) \quad \checkmark$$

$$f(x-1) + f(1-x) = x^2 + 1 + x^2 + x + 1 - x + 1 - x = 2x^2 + 1 - x \quad \checkmark$$

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$$f(x) = \sqrt{x-a} + k-1 \Rightarrow D_f = x-a \geq 0 \Rightarrow x \geq \frac{a}{f}$$

$$g(x) = \sqrt{b-ex} + k \Rightarrow D_g = b-ex \geq 0 \Rightarrow x \leq \frac{b}{e}$$

$$f \cap g = \{(1, k)\} \Rightarrow \frac{a}{e} = \frac{b}{e} = 1 \Rightarrow \boxed{a = \frac{e}{f}}, \boxed{b = e} \quad \checkmark$$

$$\left. \begin{aligned} f(x) \xrightarrow{x=1} \sqrt{e-e} + k-1 &= \boxed{k-1} \\ g(x) \xrightarrow{x=1} \sqrt{e-e} + k &= \boxed{k} \end{aligned} \right\} (f \cap g)(1) = k \Rightarrow 2(k-1) - k = k \Rightarrow \boxed{k = -1} \quad \checkmark$$

$$\frac{a}{e} - \frac{b}{e} - k = e\left(\frac{e}{e}\right) - \frac{e}{e} - (-1) = e - 1 + 1 = e \quad \checkmark$$

$$\left. \begin{aligned} f(x) &= \sqrt{m-x} + n \\ g(x) &= \sqrt{rx+r} \end{aligned} \right\} D_{f \cap g} = [-1, v], (f-g)(1) = 4\sqrt{r}$$

$$rx + r \geq 0 \Rightarrow \boxed{x \geq -1} \quad (1) \quad D_f = x \leq m \quad (2) \quad (1, 2) \Rightarrow D_{f \cap g} = [-1, m]$$

$$\boxed{m = v} \quad \checkmark$$

$$\left. \begin{aligned} f(1) &= \sqrt{v-1} + n = \boxed{1+n} \\ g(1) &= \sqrt{r(1)+r} = \boxed{\sqrt{2r}} \end{aligned} \right\} f(1) - g(1) = 4\sqrt{r} \Rightarrow 1+n - \sqrt{2r} = 4\sqrt{r} \Rightarrow \boxed{n = 1 + 5\sqrt{r}} \quad \checkmark$$

$$m+n = v + (1 + 5\sqrt{r}) \Rightarrow \boxed{v + 1 + 5\sqrt{r}} \quad \checkmark$$

$$\begin{aligned} f(n-1) &= (n-1)^r + (n-1) \\ f(1-n) &= (1-n)^r + (1-n) \end{aligned} \quad \xrightarrow{-} \quad \underline{\text{sub}}$$