

$$f(x) = \begin{cases} \sqrt{x-1} & ; [x] > c \\ \sqrt{x+1} & ; [x] \leq c \end{cases} \Rightarrow f(x) = \begin{cases} \sqrt{x-1} & ; x \geq d \\ \sqrt{x+1} & ; x < d \end{cases} \quad (1)$$

$$y = \sqrt{f(x) - x + 1} \Rightarrow f(x) - x + 1 \geq 0 \quad \begin{matrix} x \geq d \Rightarrow \sqrt{x-1} - x + 1 \geq 0 \Rightarrow x \geq 0 \quad (2) \\ x < d \Rightarrow \sqrt{x+1} - x + 1 \geq 0 \Rightarrow x \geq -\frac{1}{2} \quad (3) \end{matrix}$$

$$\begin{matrix} (1) \cap (2) \Rightarrow [x \geq d] \quad (I) \\ (2) \cap (3) \Rightarrow [-\frac{1}{2} \leq x < d] \quad (II) \end{matrix} \Rightarrow I \cup II \Rightarrow D_y = [-\frac{1}{2}, +\infty)$$

$$y = \frac{\sqrt{x} + \sqrt{x+1} + 1}{(\sqrt{x} + \sqrt{x+1})^2} = \frac{\sqrt{x} + \sqrt{x+1} + 1}{(\sqrt{x} + \sqrt{x+1})^2} = \frac{\sqrt{x} + \sqrt{x+1} + 1}{(\sqrt{x} + \sqrt{x+1})^2} = \frac{\sqrt{x} + \sqrt{x+1} + 1}{(\sqrt{x} + \sqrt{x+1})^2} \quad (4)$$

$$\Rightarrow \frac{\sqrt{x}}{\sqrt{x+1}} \quad D_y \Rightarrow \sqrt{x+1} \neq 0 \Rightarrow x \neq -\frac{1}{2} \Rightarrow D_y = \mathbb{R} - \{-\frac{1}{2}\}$$

$$D_y = (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, +\infty)$$

$$f(x) = \begin{cases} \frac{(x+1)(x^2-1)}{(x^2-1)} & ; x \neq \pm 1 \\ \frac{ax+b}{x+b} & ; x = \pm 1 \end{cases} \Rightarrow f(x) = \begin{cases} x+1 & ; x \neq \pm 1 \\ \frac{ax+b}{x+b} & ; x = \pm 1 \end{cases} \quad (5)$$

$$g(x) = x+1$$

چون $g(x) = f(x)$ بی $x=1$

$$x=1 \rightarrow 1 = \frac{a+1}{1+b} \Rightarrow a+1 = 1+b \Rightarrow a = b$$

$$x=-1 \rightarrow \frac{-a+1}{-1+b} = 1 \Rightarrow a = b-2 \quad \begin{matrix} 1+b = b-2 \Rightarrow b = -3 \\ b+2 = -3+2 = -1 \end{matrix}$$

$$g(x) = f(x) \Rightarrow \frac{(x-1)(x+1)}{(x^2+ax+b)} = \frac{(x+1)}{(x-c)} \quad (6)$$

$$x^2+ax+b = (x-1)(x-c) \Rightarrow x^2 - (c+1)x + c \Rightarrow \begin{matrix} b=c \\ a=-(c+1) \end{matrix}$$

$$f(x) = (x-c)^2(x-1)$$

$$f(x+1) = (x+1-c)^2(x+1-1) = (x-c)^2(x)$$

$$f(1-x) = (1-x-c)^2(1-x-1) = (-x-c)^2(-x) = -x(x+c)^2$$

$$g(x) = f(1+x) + f(1-x) = (x-c)^2x - x(x+c)^2$$

$$\Rightarrow x[(x^2-cx+c) - (x^2+cx+c)] = x(-2x) = -2x^2$$

$$f(1-\sqrt{x}) = \begin{cases} \sqrt{x} + 1 & x \leq 0 \\ \sqrt{x} & x > 0 \end{cases} \Rightarrow \boxed{f(-1) + f(1) = 0} \quad (8)$$

$$f(-1) \Rightarrow 1 - \sqrt{x} = -1 \Rightarrow \boxed{x = 1} \xrightarrow{x > 0} \sqrt{1} = 1$$

$$f(1) \Rightarrow 1 - \sqrt{x} = 1 \Rightarrow \boxed{x = 0} \xrightarrow{x \leq 0} \sqrt{-1} + 1 = 1$$

$$f(x + \sqrt{x}) = x^r + \sqrt{x} \Rightarrow (1)^r + \sqrt{1} = 1 + 1 = 2 \Rightarrow \boxed{x = 1} \quad (9)$$

$$f(x) \Rightarrow x + \sqrt{x} = 1 \Rightarrow \boxed{x = 0}$$

$$f(x-1) + f(1-x) = ? \quad f(x) = x^r + x$$

$$x-1 = t \Rightarrow \boxed{x = t+1} \Rightarrow f(x-1) = (t+1)^r + t+1$$

$$1-x = t \Rightarrow \boxed{x = -t+1} \Rightarrow f(1-x) = (1-x)^r + (1-x)$$

$$f(x-1) + f(1-x) = x^r + 1 + (1-x)^r + 1 - x = x^r + 1 + (1-x)^r + 1 - x = x^r + 1 + 1 - x + x^r = 2x^r + 2 - x$$

$$f(x) = \sqrt{x-a} + k-1 \Rightarrow D_f = x-a \geq 0 \Rightarrow x \geq \frac{a}{r}$$

$$g(x) = \sqrt{b-ex} + k \Rightarrow D_g = b-ex \geq 0 \Rightarrow x \leq \frac{b}{e}$$

$$f \cap g = \{(1, k)\} \Rightarrow \frac{a}{e} = \frac{b}{e} = 1 \Rightarrow \boxed{a = \frac{e}{r}}, \boxed{b = e}$$

$$\left. \begin{aligned} f(x) \xrightarrow{x=1} \sqrt{e-e} + k-1 &= \boxed{k-1} \\ g(x) \xrightarrow{x=1} \sqrt{e-e} + k &= \boxed{k} \end{aligned} \right\} (f \cap g)(1) = k \Rightarrow r(k-1) - k = k \Rightarrow \boxed{k = -1}$$

$$\frac{a}{e} - \frac{b}{e} - k = \frac{e}{e} - \frac{e}{e} - (-1) = 1 - 1 + 1 = 1 \Rightarrow \boxed{1}$$

(10)

$$\left. \begin{aligned} f(x) &= \sqrt{m-x} + n \\ g(x) &= \sqrt{rx+r} \end{aligned} \right\} D_{f \cap g} = [-1, v], (f-g)(1) = 4\sqrt{r}$$

$$rx + r \geq 0 \Rightarrow \boxed{x \geq -1} \quad (1) \quad D_f = x \leq m \quad (2) \quad (1, 2) \Rightarrow D_{f \cap g} = [-1, m] \Rightarrow \boxed{m = v}$$

$$\left. \begin{aligned} f(1) &= \sqrt{v-1} + n = \boxed{1+n} \\ g(1) &= \sqrt{r(1)+r} = \boxed{\sqrt{2r}} \end{aligned} \right\} f(1) - g(1) = 4\sqrt{r} \Rightarrow 1+n - \sqrt{2r} = 4\sqrt{r} \Rightarrow \boxed{n = 1 + 3\sqrt{r}}$$

$$m+n = v + (1 + 3\sqrt{r}) \Rightarrow \boxed{m = v - 1 - 3\sqrt{r}}$$