

۲۰ عالی نوشتن

$$f(n) = \begin{cases} 2n-1, & [n] > 4 \\ 4n+3, & [n] \leq 4 \end{cases} \rightarrow f(n) = \begin{cases} 2n-1, & n \geq 5 \\ 4n+3, & n < 5 \end{cases}$$

$$y = \sqrt{f(n) - n + 1} \Rightarrow f(n) - n + 1 \geq 0 \begin{cases} 2n-1-n+1 \geq 0 \Rightarrow n \geq 0 \rightarrow n \geq 5 \Rightarrow n \geq 5 \quad \text{I} \\ 4n+3-n+1 \geq 0 \Rightarrow 3n+4 \geq 0 \Rightarrow 3n \geq -4 \Rightarrow n \geq -\frac{4}{3} \rightarrow n \leq 4 \quad \text{II} \end{cases}$$

II - $-\frac{4}{3} \leq n < 5$

I, II $\rightarrow D_y = [-\frac{4}{3}, 4]$

$$y = \frac{2n^2 + 4n + 3}{(4n+3)^2} \rightarrow y = \frac{2(1n^2 + 2n + 1)}{(2n+1)^2} = \frac{2(2n+1)}{(2n+1)^2} = \frac{2}{2n+1}$$

$$2n+1 \neq 0 \Rightarrow 2n \neq -1 \Rightarrow n \neq -\frac{1}{2} \rightarrow D_y = \mathbb{R} \setminus \{-\frac{1}{2}\}$$

$$f(n) = g(n) \rightarrow \frac{n^2 + 2n^2 - n - 2}{n^2 - 1} = x + C \Rightarrow \frac{(n-1)(n+1)(n+2)}{(n-1)(n+1)} = x + C \Rightarrow C = 1$$

$$\frac{ax+2}{x+b} = x+C \begin{cases} (n=1) \Rightarrow \frac{a+2}{1+b} = 1+2 \rightarrow 3+3b = a+2 \Rightarrow 3b-a = -1 \\ (n=-1) \Rightarrow \frac{-a+2}{b-1} = -1+2 \rightarrow b-1 = -a+2 \Rightarrow b+a = 3 \end{cases}$$

$$\begin{cases} 3b-a = -1 \\ b+a = 3 \end{cases} \Rightarrow 4b = 2 \Rightarrow b = \frac{1}{2}$$

$$b+C = \frac{1}{2} + 2 = \frac{5}{2}$$

$$f(n) = \frac{n+1}{n-C} \quad g(n) = \frac{n^2-1}{n^2+an+b} = \frac{(n-1)(n+1)}{(n-1)(n+1)}$$

$$\frac{(n-1)(n+1)}{(n-1)(n+1)} = \frac{n+1}{n-1} \Rightarrow C = 1$$

$$\begin{cases} a = -2 \\ b = 1 \end{cases}$$

$$a+b+C = -2+1+1 = 0$$

$$f(n) = (n-2)^2 (n-1)$$

$$g(n) = f(n+1) + f(1-n) \rightarrow (n+1-2)^2 (n+1-1) + (1-n-2)^2 (1-n-1) = (n-1)^2 (n) + (-n-2)^2 (-n)$$

$$\rightarrow g(n) = n^3 - 2n^2 + n - n^3 - 4n^2 - 4n = -3n^2 - 3n$$

$$g(n) = -3n^2$$

$$f(1-x) = \begin{cases} \sqrt{x} + 1 & x \leq 0 \\ \sqrt{x} & x > 0 \end{cases} \quad f(-1) + f(1) = ?$$

$$\left. \begin{aligned} f(-1) = f(1-x) \Rightarrow -1 = 1-x \Rightarrow x = 2 \rightarrow \sqrt{2} = 1 \\ f(1) = f(1-x) \Rightarrow 1 = 1-x \Rightarrow x = 0 \rightarrow \sqrt{0} + 1 = 1 \end{aligned} \right\} f(-1) + f(1) = 1 + 1 = 2$$

$$f(4) = ?$$

$$f(x + \sqrt{x}) = x^2 + 4x$$

$$f(m + \sqrt{m}) = f(4) \Rightarrow 4 = m + \sqrt{m} \xrightarrow{\sqrt{m}=t} t^2 + t - 4 = 0 \Rightarrow (t+4)(t-1) = 0 \quad \begin{cases} t = \sqrt{m} = -4 \text{ ođđ} \\ t = \sqrt{m} = 1 \Rightarrow m = 1 \end{cases}$$

$$f(4) = 4^2 + 4(4) = 16 + 16 = 32$$

$$f(n) = n^2 + n = n(n+1)$$

$$f(n-1) + f(1-n) = [(n-1)(n)(n+1)] + [(-n+1)(-n+2)(-n)] = 0$$

$$f(x) = \sqrt{bx - a} + k - 1 \rightarrow D_f = \{x \mid bx - a \geq 0\} \Rightarrow \{x \mid x \geq \frac{a}{b}\} \quad \begin{cases} \frac{a}{b} = 1 \Rightarrow a = 2 \Rightarrow a = \frac{2}{b} \\ \frac{b}{2} = 1 \Rightarrow b = 2 \end{cases}$$

$$g(x) = \sqrt{bx - a} + k \rightarrow D_g = \{x \mid bx - a \geq 0\} \Rightarrow \{x \mid x \geq \frac{a}{b}\}$$

$$\Rightarrow f(x) = \sqrt{bx - a} + k - 1 \quad g(x) = \sqrt{bx - a} + k$$

$$f(1) - g(1) = 2k - 2 - k = -k - 2 \Rightarrow -k - 2 = k \Rightarrow k = -1$$

$$2a - \frac{b}{2} - k = 2 \times \frac{2}{2} - \frac{2}{2} - (-1) = 2 - 1 + 1 = 2$$

$$D_{f \wedge g} = D_f \cap D_g = [-1, 5]$$

$$D_g \Rightarrow x + 1 \geq 0 \rightarrow x \geq -1 \rightarrow D_g = [-1, +\infty)$$

$$D_f \Rightarrow m - x \geq 0 \rightarrow x \leq m \rightarrow m = \sqrt{5} \quad D_f = (-\infty, \sqrt{5}]$$

$$f - g(x) = 4\sqrt{x} \rightarrow \sqrt{5 - x} + m - \sqrt{4 + x} = 4\sqrt{x} \Rightarrow m = 4\sqrt{x} + \sqrt{4 + x} - \sqrt{5 - x}$$

$$m + n = 5 + 4\sqrt{5} - 2 = 3 + 4\sqrt{5}$$