

$$f(n) = \begin{cases} 2n-1, & [n] > \varepsilon \\ \varepsilon n + 2, & [n] \leq \varepsilon \end{cases} \rightarrow f(n) = \begin{cases} 2n-1, & n > \varepsilon \\ \varepsilon n + 2, & n \leq \varepsilon \end{cases}$$

$$y = \sqrt{f(n) - n + 1} \Rightarrow f(n) - n + 1 \geq 0 \begin{cases} 2n-1-n+1 \geq 0 \Rightarrow n \geq 0 \Rightarrow n \geq 0 \Rightarrow n \geq 0 \quad \text{I} \\ \varepsilon n + 2 - n + 1 \geq 0 \Rightarrow \varepsilon n + 3 \geq n \Rightarrow \varepsilon n - n \geq -3 \Rightarrow n \leq \frac{3}{1-\varepsilon} \quad \text{II} \end{cases}$$

$$\text{I, II} \rightarrow D_y = \left[-\frac{\varepsilon}{1-\varepsilon}, +\infty\right) \quad \text{III} - \frac{\varepsilon}{1-\varepsilon} \leq n \leq \frac{3}{1-\varepsilon}$$

$$y = \frac{2\varepsilon n^2 + 4n + 1 + \varepsilon n + 2}{(\varepsilon n + \varepsilon n + 1)^2} \rightarrow y = \frac{2(1-\varepsilon)n^2 + 4n + 1}{(\varepsilon n + 1)^2} = \frac{2(2n+1)^2}{(\varepsilon n + 1)^2} = \left(\frac{2}{\varepsilon n + 1}\right)^2$$

$$\varepsilon n + 1 \neq 0 \Rightarrow \varepsilon n \neq -1 \Rightarrow n \neq -\frac{1}{\varepsilon} \rightarrow D_y = \mathbb{R} \setminus \left\{-\frac{1}{\varepsilon}\right\}$$

$$f(n) = g(n) \rightarrow \frac{n^2 + \varepsilon n^2 - n - 2}{n^2 - 1} = x + C \Rightarrow \frac{(n-1)(n+1)(n+2)}{(n-1)(n+1)} = x + C \Rightarrow C = 2$$

$$\frac{ax + 2}{x + b} = x + C \begin{cases} (n=1) \Rightarrow \frac{a+2}{1+b} = 1+2 \rightarrow 3+3b = a+2 \Rightarrow 3b - a = -1 \\ (n=-1) \Rightarrow \frac{-a+2}{b-1} = -1+2 \rightarrow b-1 = -a+2 \Rightarrow b+a = 3 \end{cases}$$

$$\begin{cases} 3b - a = -1 \\ b + a = 3 \end{cases} \Rightarrow \begin{cases} 3b - a = -1 \\ 3b + a = 3 \end{cases} \Rightarrow \begin{cases} 6b = 2 \Rightarrow b = \frac{1}{3} \\ a = 3 - b = \frac{8}{3} \end{cases}$$

$$b + C = \frac{1}{3} + 2 = \frac{7}{3} \in \mathbb{R}$$

$$f(n) = \frac{n+1}{n-C} \quad g(n) = \frac{n^2-1}{n^2+an+b} = \frac{(n-1)(n+1)}{(n-1)(n+1)}$$

$$\frac{(n-1)(n+1)}{(n-1)(n+1)} = \frac{n+1}{n-1} \Rightarrow C = 1$$

$$\begin{cases} a = -2 \\ b = 1 \end{cases}$$

$$a+b+C = -2+1+1 = 0$$

برای اینکه دو تابع برابر باشند باید دامنه هر دو برابر باشند و
بنابراین هر x دامنه داشته باشد $f(n) = g(n)$

$$f(n) = (n-2)^2 (n-1)$$

$$g(n) = f(n+1) + f(1-n) \rightarrow (n+1-2)^2 (n+1-1) + (1-n-2)^2 (1-n-1) = (n-1)^2 (n) + (-n-1)^2 (-n)$$

$$\rightarrow g(n) = \cancel{n^3} - \varepsilon n^2 + \varepsilon n \rightarrow \cancel{n^3} - \varepsilon n^2 - \varepsilon n = -\varepsilon n^2$$

$$g(n) = -\varepsilon n^2$$

$$f(1-x) = \begin{cases} \sqrt{x} + 1 & x \leq 0 \\ \sqrt{x} & x > 0 \end{cases} \quad f(-1) + f(x) = ?$$

$$\left. \begin{aligned} f(-1) &= f(1-x) \Rightarrow -1 = 1-x \Rightarrow x = 2 \rightarrow \sqrt{2} = f \\ f(x) &= f(1-x) \Rightarrow x = 1-x \Rightarrow x = -1 \rightarrow \sqrt{-1} + 1 = 1 \end{aligned} \right\} f(-1) + f(x) = 1 + 1 = \textcircled{2}$$

$$f(4) = ?$$

$$f(x + \sqrt{x}) = x^2 + 4x$$

$$f(x + \sqrt{x}) = f(4) \Rightarrow 4 = x + \sqrt{x} \xrightarrow{\sqrt{x}=t} t^2 + t - 4 = 0 \Rightarrow (t+4)(t-1) = 0 \left\{ \begin{aligned} t = \sqrt{x} = -4 \text{ (reject)} \\ t = \sqrt{x} = 1 \Rightarrow x = 1 \end{aligned} \right.$$

$$f(4) = 1^2 + 4(1) = 1 + 4 = \textcircled{5}$$

$$f(n) = n^2 + n = n(n+1)$$

$$f(n-1) + f(1-n) = [(n-1)(n)(n+1)] + [(-n+1)(-n+2)(-n)] = \textcircled{0}$$

$$f(x) = \sqrt{bx - a} + k - 1 \rightarrow D_f = [n, \infty) \Rightarrow \begin{cases} \frac{a}{b} = 1 \Rightarrow a = b \\ \frac{b}{b} = 1 \Rightarrow b = 1 \end{cases} \Rightarrow a = 1, b = 1$$

$$g(x) = \sqrt{b - ax} + k \rightarrow D_g = [0, b/a] \Rightarrow \begin{cases} \frac{a}{b} = 1 \Rightarrow a = b \\ \frac{b}{b} = 1 \Rightarrow b = 1 \end{cases} \Rightarrow a = 1, b = 1$$

$$\Rightarrow f(x) = \sqrt{x - 1} + k - 1 \quad g(x) = \sqrt{1 - x} + k$$

$$f(1) - g(1) = k - 1 - k = -1 \Rightarrow -1 = k \Rightarrow k = -1$$

$$f(a) - \frac{b}{a} - k = \sqrt{a - 1} - \frac{1}{a} - (-1) = \sqrt{a - 1} + 1 - \frac{1}{a} = \textcircled{0}$$

$$D_{f \wedge g} = D_f \cap D_g = [-1, 1]$$

$$D_g \Rightarrow x + 1 \geq 0 \rightarrow x \geq -1 \rightarrow D_g = [-1, +\infty)$$

$$D_f \Rightarrow m - x \geq 0 \rightarrow x \leq m \rightarrow \boxed{m = 1} \quad D_f = (-\infty, 1]$$

$$f - g(x) = 4\sqrt{x} \rightarrow \sqrt{x - 1} + m - \sqrt{1 - x} = 4\sqrt{x} \Rightarrow m = \sqrt{x - 1} + 4\sqrt{x} - \sqrt{1 - x}$$

$$m + n = 1 + \sqrt{x - 1} - \sqrt{1 - x} = \textcircled{1 + \sqrt{x - 1}}$$