

مکرمه ۲۳

بابان شریفی

تالیف شماره ۳

سوال ۱:

$$f(-x) - xf(x) = 2|x| + 1 \xrightarrow{x} xf(-x) - x^2 f(x) = 2|x| + 1$$

$$-x^2 f(x) + xf(-x) = 2(1-x) + 1 \xrightarrow{x} -f(x) - xf(-x) = -2|x| - 1$$

$$f(x)(-x^2 - 1) = 2|x| + 1 - 2|x|$$

$$x > 0 \rightarrow f(x) = \frac{2x^2 - x - 1}{-x^2 - 1} \quad x < 0 \rightarrow \frac{-2x^2 + 3x - 1}{-x^2 - 1}$$

سوال ۲:

$$f(x) + f(0) = x^2 - 3x + 4$$

$$2 \rightarrow f(0) = 0 - 3 \cdot 0 + 4 - f(0)$$

$$x=0 \rightarrow 2 \rightarrow 2f(0) = 4 \rightarrow f(0) = 2$$

$$f(x) = x^2 - 3x + 3 \rightarrow f(x^2) = x^4 - 3x^2 + 3$$

سوال ۳:

$$y = \frac{x^2 + x + 2}{x-1} \rightarrow 2x - y = x^2 + x + 2$$

$$x^2 - 2x + y + x + 2 = 0 \rightarrow x^2 + (-2+y)x + (y+2) = 0$$

$$x = \frac{y-1 \pm \sqrt{y^2 - 4y - 4}}{2}$$

$$2 \rightarrow R_f = \mathbb{R} - (-1, 1)$$

$$\Delta = (1-y)^2 - 4(y+2) = 1 - 2y + y^2 - 4y - 8 = y^2 - 6y - 7$$

$$\frac{-1}{+} \quad \frac{7}{-} \quad \frac{+}{+}$$

$$(y-7)(y+1)$$

سوال ۴:

$$الف) y = 2x^2 - 4x + 1 \rightarrow x_2 = \frac{-b}{2a} = \frac{2}{4} = \frac{1}{2} \rightarrow y_2 = 2 \cdot \frac{1}{4} - \frac{4 \cdot \frac{1}{2}}{4} + 1 = \frac{1}{2} - 1 + 1 = \frac{1}{2}$$

صورتی

$$U \rightarrow R = \left[-\frac{1}{2}, +\infty\right) \quad -\frac{\infty}{4} \rightarrow -\frac{3}{4} \cdot 2 = -\frac{3}{2}$$

$$ب) y = \sqrt{-x^2 + 4x - 4} \rightarrow \text{...} \quad x_2 = \frac{-4}{-2} = 2 \rightarrow y = -4 + 1 + 4 = 1$$

$$\sqrt{(-\infty, 4]} = [0, 2]$$

$$ج) y = \sqrt{x^2 - 3x + 1} \rightarrow [0, +\infty)$$

$$د) y \in \log(x^2 - 4x + 7x + 4) \rightarrow (-\infty, +\infty)$$

الف) $y = \frac{3x+1}{2x-4} \rightarrow IR - \left\{ \frac{3}{2} \right\}$ سوال ٥

معاين افقى

ب) $y = \sqrt{\frac{3x+1}{x-2}} \sim [2, +\infty) - \left\{ \frac{3}{2} \right\}$

$\sqrt{3} = 2$

ج) $y = \frac{x^2+1}{\sqrt{x^2+3}} = \frac{x^2+3}{\sqrt{x^2+3}} + \frac{1}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \sim \frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

$\min = \sqrt{3} \rightarrow \frac{4\sqrt{3}}{3}$

$R = \left[\frac{4\sqrt{3}}{3}, +\infty \right)$

$y = x^2 + \frac{1}{x^2+1} \sim x^2 + \frac{1}{x^2+1} - 1 = \frac{1}{x^2+1} - 1 = \frac{1}{x^2+1} - \frac{x^2+1}{x^2+1} = \frac{1-x^2-1}{x^2+1} = \frac{-x^2}{x^2+1}$

$\min x^2$

$R = \left[\frac{1}{2}, +\infty \right)$

$y = \frac{3x^2+1}{x^2-1} \sim \frac{3x^2+1}{x^2-1} - 3 = \frac{3x^2+1-3x^2+3}{x^2-1} = \frac{4}{x^2-1}$

$\min x \rightarrow 0 \rightarrow \frac{3 \cdot 0 + 1}{0-1} = -1$

$\max x \rightarrow +\infty \rightarrow \frac{3(\infty)+1}{(\infty)-1} = 3$

$R = IR - (-1, 3)$ سوال ٦

$2x^2 - y = 3x^2 - yx^2 + 1 \rightarrow 3x^2 - yx^2 + 1 = 0$ (-2^2 - 2 + 12)

$x^2(3-y) + 1 = 0 \rightarrow \Delta = 0 - 4(3-y)(3-y) = y^2 + y - 12$

$2x/y = \frac{0 \pm \sqrt{y^2+y-12}}{2} \sim \frac{\pm \sqrt{y^2+y-12}}{2}$ (2+3)(2-3)

$\frac{1}{4} \mid - \mid +$

$R = (-1, 3)$

$f(x) = \frac{1 \sin x + 1}{\sin x - 1} \sim \frac{1+1}{1-1} = -1$ سوال ٧

$\frac{1}{1-1} = -1$

$\frac{-1+1}{-1-1} = \frac{0}{-2} = 0$

$R = \left[-1, \frac{1}{2} \right]$

$y = \left| \frac{3x-1}{x} \right|$ سوال ٨

$\frac{3}{x} = \text{افقى}$

$\lim_{x \rightarrow 0} = \pm \infty$

$R = [0, +\infty)$

$y = \sqrt{\frac{9x-1}{x}} \sim \sqrt{\frac{9(x-1)}{x}} = \sqrt{\frac{9}{x}} = \frac{3}{\sqrt{x}} = R$

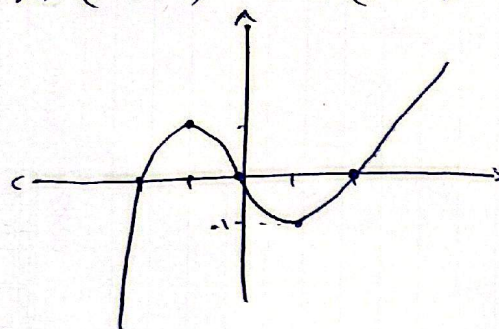
$x = 1$

$$y = x|x| - 2x$$

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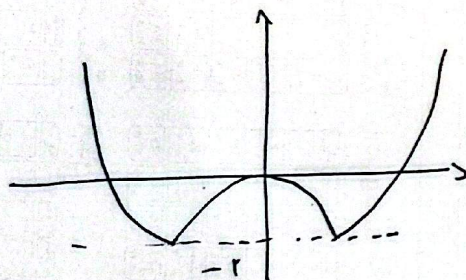
$x < 0$	$x \geq 0$
$-x^2 - 2x$	$x^2 - 2x \rightarrow (1, -1)$
$-x(x+2)$	$x(x-2)$

سوال 9:



$$\mathbb{R} = \mathbb{R}$$

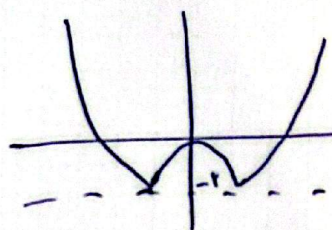
$$f(x) =$$



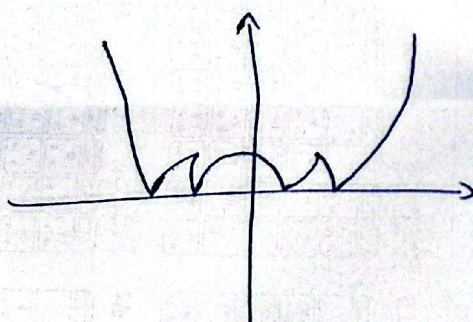
$$y = |f(|x| + 1)|$$

سوال 10:

$$2, f(|x|) =$$



$$\rightarrow y = f(|x| + 1)$$



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