

$$f(-n) - n f(n) = 2|n| + 1 \xrightarrow{n \geq 0} f(-n) - n f(n) = 2n + 1$$

$$\xrightarrow{n \leq 0} f(-n) - n f(n) = 2|-n| + 1 \xrightarrow{x=n}$$

-1
(1)

$$f(-n) - n f(n) = 2n + 1$$

$$n f(-n) - n^2 f(n) = 2n^2 + n$$

$$\frac{f(-n) - n^2 f(n)}{f(-n)(1+n^2)} = \frac{2n^2 + 3n + 1}{1+n^2} \Rightarrow f(-n) = \frac{2n^2 + 3n + 1}{1+n^2}$$

برای $n < 0$

$$f(n) = \frac{2n^2 - 3n + 1}{1+n^2}$$

$n > 0$ هم می توانیم بکنیم و به دست می آید
صفر افتد...

$$f(n) + f(0) = n^2 - 3n + 4 \xrightarrow{n=0} f(0) + f(0) = 4 \rightarrow f(0) = 2 \Rightarrow f(n) = n^2 - 3n + 4 - f(0)$$

$$\Rightarrow f(n) = n^2 - 3n + 2$$

(2) -2

$$y = \frac{n^2 + n + 2}{n-1} \rightarrow y_n - y = n^2 + n + 2 \Rightarrow n^2 + n(1-y) + y + 2 = 0 \rightarrow$$

(2) -3

$$\Delta = (1-y)^2 - 4(1)(y+2) = y^2 - 2y + 1 - 4y - 8 = y^2 - 6y - 7 \geq 0$$

$$n_1, n_2 = \frac{-1+y \pm \sqrt{y^2 - 6y - 7}}{1}$$

$$(y-7)(y+1) \geq 0 \quad \frac{-1}{+} \frac{7}{-} \frac{1}{+}$$

$$R_f = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف) } y = \frac{2n^2 - 2n + 1}{n^2} \text{ انت } \left| \frac{-b}{2a} \right| = \left| \frac{\frac{2}{n^2}}{-\frac{2}{n}} \right| = \left| -\frac{1}{n} \right| \Rightarrow R_f = \left[-\frac{1}{n}, +\infty \right)$$

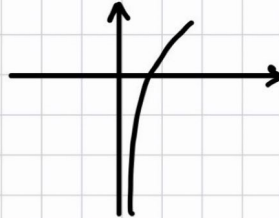
$$2\left(\frac{2}{n^2}\right)^2 - 2\left(\frac{2}{n^2}\right) + 1 = -\frac{1}{n}$$

(2) -4

$$\text{ب) } y = \sqrt{n^2 + 4n - 4} \rightarrow \text{انت } \left| \frac{4}{2} \right| = 2 \rightarrow \sqrt{(-\infty, 4]} = R_f = [0, 4]$$

$$\text{ج) } y = \sqrt{n^2 - 3n^2 + 4n + 1} \rightarrow \sqrt{(-\infty, +\infty)} = [0, +\infty)$$

د) $\log(x^3 - 4x^2 + 7x + K) \rightarrow R_f = \mathbb{R}$



الف) $y = \frac{4x+1}{x-4}$ تابع معکوس
 $R_f = \mathbb{R} - \left\{\frac{4}{3}\right\}$ ایضاً ۲ - ۵

ب) $y = \sqrt{\frac{4x+1}{x-4}} \rightarrow \sqrt{\mathbb{R} - \frac{4}{3}} = [0, +\infty) - \left\{\frac{2}{\sqrt{3}}\right\}$

ج) $y = \frac{x^2+4}{\sqrt{x^2+4}} = \frac{x^2+4}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \xrightarrow{\text{مقدار معکوس}} \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \Rightarrow R_f = \left[\sqrt{4} + \frac{1}{\sqrt{4}}, +\infty\right) = \left[\frac{5}{2}, +\infty\right)$

$\left[\frac{5}{2}, +\infty\right)$

د) $y = x^2 + \frac{1}{x^2+4} \rightarrow +\infty - 4 \rightarrow x^2 + 4 + \frac{1}{x^2+4} \xrightarrow{\text{مقدار معکوس}} \frac{1}{x^2+4} - 4 = \frac{1}{x^2+4} \rightarrow R_f = \left[\frac{1}{4}, +\infty\right)$

$y = \frac{3x^2+4}{x^2-1} \xrightarrow{\text{دریافت}} x^2 = -\frac{-y-4}{y-3} = \frac{y+4}{y-3} \rightarrow x = \pm \sqrt{\frac{y+4}{y-3}} + \frac{-4}{y-3} + \frac{3}{y-3}$ ۲

$R_f = (-\infty, -4] \cup (3, +\infty)$ $y x^2 - y = 3x^2 + 4 \rightarrow x^2(y-3) = 4+y \rightarrow x^2 = \frac{y+4}{y-3}$

$x=0 \rightarrow y = \frac{4}{-1} = -4$

$x \rightarrow +\infty \rightarrow y = \frac{3(+\infty)}{1(+\infty)} = 3$

$\rightarrow (-\infty, -4] \cup (3, +\infty)$ درست است

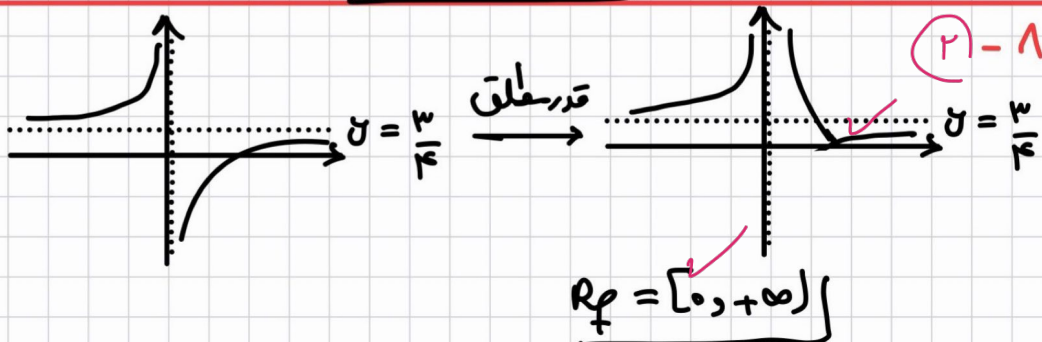
$\sin x \rightarrow -1, 1$

$\sin x = 1 \rightarrow y = \frac{2(1)+4}{1-3} = \frac{6}{-2} = -3$ ۲ - ۷

$\sin x = -1 \rightarrow y = \frac{2(-1)+4}{-1-3} = \frac{2}{-4} = -\frac{1}{2}$

$\Rightarrow R_f = \left[-\frac{1}{2}, -3\right]$

الف) $y = \left|\frac{3x-1}{x}\right|$

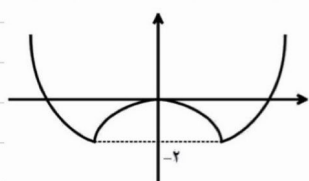
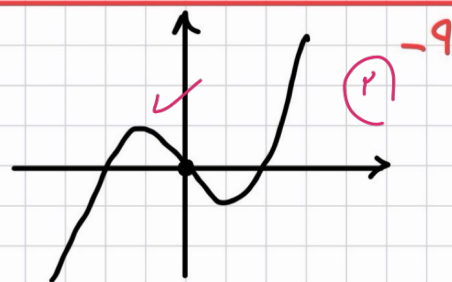


$$b) y = \sqrt{\frac{9n-18}{4n-18}} = \sqrt{\frac{9(n-2)}{4(n-2)}} = \sqrt{\frac{9}{4}} = \frac{3}{2} \rightarrow R_f = \left\{ \frac{3}{2} \right\}$$

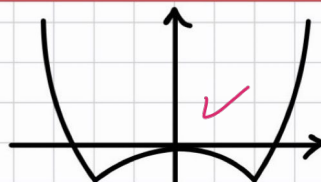
$$y = n|n| - r n$$

$$f(n) \begin{cases} n^2 - r n & n \geq 0 \\ -n^2 - r n & n < 0 \end{cases}$$

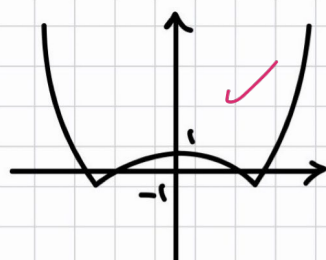
$$R_f = \mathbb{R}$$



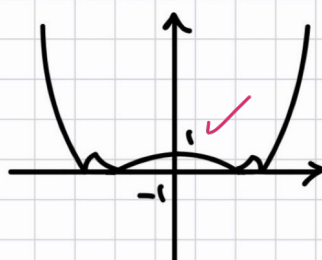
$$|f(n)| + 1 \rightarrow f(n)$$



$$f(n) + 1$$



$$|f(n)| + 1$$



تمام تلاشم رو کردم شکل حائز
در بیا ریختید آه خوب نشد ☹️

۱- جای n و $-n$ قرار می دهیم و یک دستگاه می سازیم :

$$\begin{aligned} f(-n) - n f(n) &= r|n| + 1 \xrightarrow{\times n} \begin{cases} n f(-n) - n^2 f(n) = r n|n| + n & (I) \\ f(n) + n f(-n) = r|n| + 1 & (II) \end{cases} \\ f(n) + n f(-n) &= r|n| + 1 \end{aligned}$$

$$\underline{II - I} \rightarrow (1 + n^2) f(n) = r(1 - n)|n| + 1 - n \Rightarrow f(n) = \frac{(1 - n)(r|n| + 1)}{1 + n^2}$$