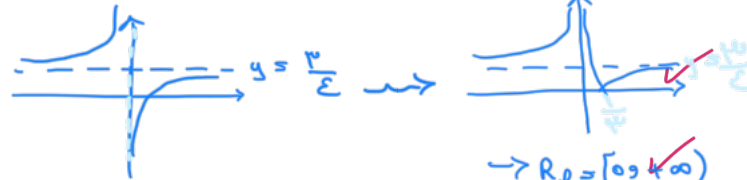
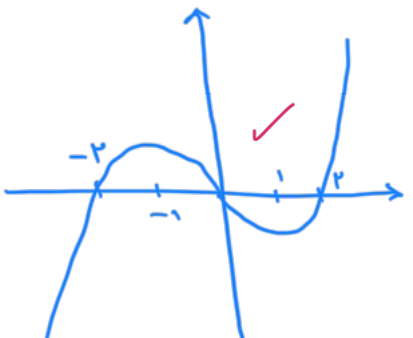
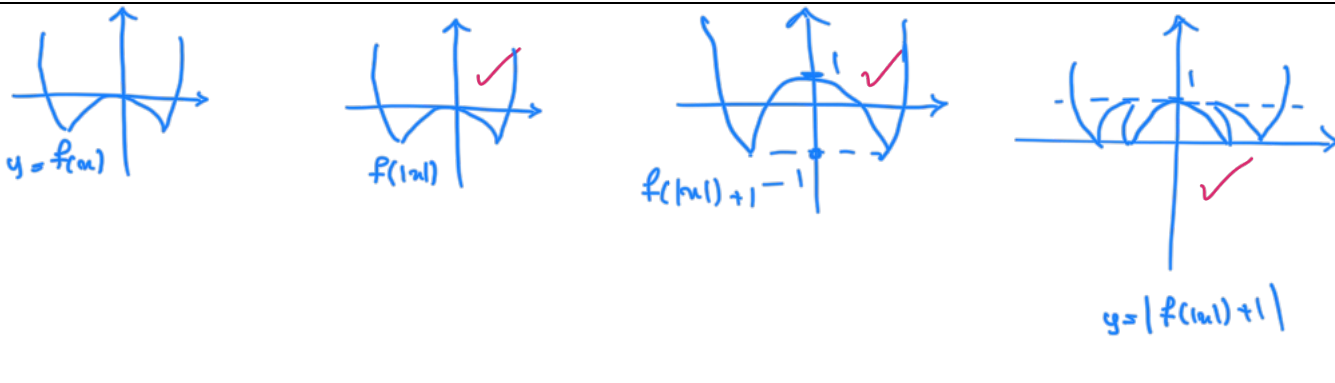


۱۹

$\begin{cases} f(-n) - nf(n) = 2 n + 1 \\ (f(n) - (-n)f(-n) = 2(1-n) + 1) \times n \end{cases}$ $(n^2 + 1)f(-n) = 2n^2 + 3n + 1 \rightarrow f(-n) = \frac{2n^2 + 3n + 1}{n^2 + 1}$ $\rightarrow f(n) = \frac{2n^2 - 3n + 1}{n^2 + 1} \checkmark$	نصفه کنین > چه ت ۱
$f(n) + f(0) = n^2 - 3n + 4 \xrightarrow{n=0} 2f(0) = 4 \rightarrow f(0) = 2$ $f(n) + 2 = n^2 - 3n + 4 \rightarrow f(n) = n^2 - 3n + 2$ $\rightarrow f(n^2) = n^4 - 3n^2 + 2 \checkmark$	۲
$y = \frac{n^2 + n + 2}{n - 1} \rightarrow y(n - 1) = n^2 + n + 2 \rightarrow n^2 - yn + n + y + 2 = 0 \rightarrow n^2 + (1 - y)n + y + 2 = 0$ $\rightarrow n = \frac{y + 1 \pm \sqrt{\Delta}}{2}$ $\rightarrow (-y + 1)^2 - 4(y + 2) \geq 0 \rightarrow (y - 7)(y + 1) \geq 0$ $\begin{array}{c c c} -1 & 7 & \\ \hline + & - & + \end{array} \rightarrow R_f = (-\infty, -1] \cup [7, +\infty)$	۳
الف) $y = 2n^2 - 5n + 1 \rightarrow \min \left n = \frac{5}{4} \right y = 2 \times \frac{25}{16} - 5 \times \frac{5}{4} + 1 = -\frac{19}{4} \rightarrow R_f = [-\frac{19}{4}, +\infty)$ ب) $y = \sqrt{-n^2 + 4n - 5} \rightarrow \max \left \varepsilon \rightarrow \sqrt{(-\infty, \varepsilon]} \rightarrow R_f = [0, 2] \right$ ج) $\sqrt{(-\infty, +\infty)} \rightarrow R_f = [0, +\infty)$ د) $\log(-\infty, +\infty) \rightarrow \log(0, +\infty) \rightarrow R_f = (-\infty, +\infty)$ 	۴
الف) $R_f = \mathbb{R} - \{\frac{1}{2}\}$ ب) $\sqrt{\mathbb{R} - \{\varepsilon\}} \rightarrow \mathbb{R}_f = [0, +\infty) \cup \{\frac{1}{2}\}$ ج) $\frac{n^2 + 3n + 1}{\sqrt{n^2 + 3n}} = \sqrt{n^2 + 3n} + \frac{1}{\sqrt{n^2 + 3n}}$ $\min = \sqrt{3} \rightarrow \mathbb{R}_f = [\sqrt{3} + \frac{1}{\sqrt{3}}, +\infty) \cup [\frac{\varepsilon}{\sqrt{3}}, +\infty)$ د) $\frac{n^2 + 1}{n^2 + \varepsilon} + \varepsilon - \varepsilon$ $\min = \varepsilon \rightarrow \varepsilon + \frac{1}{\varepsilon} = \frac{19}{\varepsilon}$ $\rightarrow R_f = [\frac{19}{\varepsilon} - \varepsilon, +\infty) = [\frac{1}{\varepsilon}, +\infty)$	۵

$y = \frac{\mu x^2 + \varepsilon}{x^2 + 1} \rightarrow \mu x^2 = \frac{-y - \varepsilon}{y - \mu} = \frac{y + \varepsilon}{y - \mu} \rightarrow x = \pm \sqrt{\frac{y + \varepsilon}{y - \mu}} \geq 0 \rightarrow \frac{-\varepsilon - \mu}{\mu - \frac{\mu}{\mu} + 1}$ $IR_f = (-\infty, -\varepsilon] \cup (\mu, +\infty)$	(2) 6
$y = \frac{\mu x^2 + \varepsilon}{x^2 - 1} \xrightarrow{x^2=0} y = \frac{\varepsilon}{-1} = -\varepsilon$ $\xrightarrow{x^2 \rightarrow +\infty} y = \frac{\mu(+\infty)}{(+\infty)} = \mu$ $IR_f = (-\infty, -\varepsilon] \cup (\mu, +\infty)$	(2) 7
الف) $y = \frac{\mu x - 1}{\varepsilon}$  $\rightarrow R_f = [0, +\infty)$ ب) $y = \sqrt{\frac{a x - 1}{\varepsilon a - 1}}$ $\xrightarrow{x=0} y = \sqrt{\frac{a(0-1)}{\varepsilon(a-1)}} \rightarrow x \neq -\frac{1}{a} \rightarrow y = \frac{\mu}{\varepsilon} \rightarrow R_f = \left\{ \frac{\mu}{\varepsilon} \right\}$	(2) 8
$y = x x - \mu x \xrightarrow{x \geq 0} y = x^2 - \mu x$ $\xrightarrow{x \leq 0} y = -x^2 - \mu x$  $R_f = \mathbb{R}$	(2) 9
 $y = f(x) + 1$	(2) 10

۱- جای n ، $-n$ قرار می دهیم و یک دستگاه معادلات می یابیم:

$$\begin{aligned} f(-n) - n f(n) &= 2|n| + 1 \xrightarrow{\times n} \begin{cases} n f(-n) - n^2 f(n) = 2n|n| + n & \text{(I)} \\ f(n) + n f(-n) = 2|n| + 1 & \text{(II)} \end{cases} \\ f(n) + n f(-n) &= 2|n| + 1 \end{aligned}$$

$$\underline{\text{II} - \text{I}} \rightarrow (1 + n^2) f(n) = 2(1 - n)|n| + 1 - n \Rightarrow f(n) = \frac{(1 - n)(2|n| + 1)}{1 + n^2}$$