

هستی صدری / دوازدهم رشته B / آرد تکلیف

$$1) \begin{aligned} f(-x) - x f(x) &= 2|x| + 1 \xrightarrow{x=1} x f(-x) - x^2 f(x) = 2x|x| + 1 \\ f(x) + x f(-x) &= 2|x| + 1 \end{aligned} \quad \left. \begin{aligned} x f(-x) - x^2 f(x) &= 2x|x| + 1 \\ f(x) + x f(-x) &= 2|x| + 1 \end{aligned} \right\} \begin{aligned} f(x) + x f(-x) &= 2|x| + 1 \\ f(x) + x f(-x) &= 2|x| + 1 \end{aligned}$$

$$f(x) + x^2 f(x) = -2x|x| - x + 2|x| + 1 \rightarrow f(x) = \frac{(2|x|+1)(1-x)}{1+x^2}$$

$$2) \begin{aligned} f(x) + f(0) &= x^2 - 3x + 4 \xrightarrow{x=0} 2f(0) = 4 \\ \rightarrow f(0) &= 2 \end{aligned}$$

$$\rightarrow f(x) = x^2 - 3x + 4 \rightarrow f(x^2) = x^4 - 3x^2 + 4$$

$$3) \begin{aligned} x - y &= x^2 + x + 2 \rightarrow x^2 + x - y + 2 + y = 0 \rightarrow \Delta = (1-y)^2 - 4(2+y) \\ &= 1 - 2y + y^2 - 8 - 4y = y^2 - 6y - 7 \\ &= (y-7)(y+1) > 0 \end{aligned}$$

$$y < -1 \quad \text{or} \quad y > 7$$

$$x = \frac{-(1-y) \pm \sqrt{y^2 - 6y - 7}}{2}$$

$$4) \text{ الف) } y = 2x^2 - 5x + 1 \rightarrow \frac{-\Delta}{4a} = \frac{-(25-4)}{4} = -\frac{11}{4} \rightarrow R = [-\frac{11}{4}, +\infty)$$

$$ب) y = \sqrt{-x^2 + 4x - 5} \rightarrow \frac{-\Delta}{4a} = \frac{14}{-4} = -\frac{7}{2} \quad R \in [-\frac{7}{2}, 2]$$

$$ج) \sqrt{x^2 - 3x^2 + 4x + 1} \rightarrow R \in [0, \infty)$$

$$د) \log(x^3 - 4x^2 + 7x + 4) \rightarrow x^3 - 4x^2 + 7x + 4 > 0 \quad R = \mathbb{R}$$

$$(x-4)(x^2 - 2x - 1)$$

$$5) \text{ الف) } y = \frac{3x+1}{2x-4} \rightarrow 2xy - 4y = 3x+1 \rightarrow x(2y-3) = 4y+1$$

$$\rightarrow x = \frac{4y+1}{2y-3} \quad R \Rightarrow \mathbb{R} - \left\{ \frac{3}{2} \right\}$$

$$ب) y = \sqrt{\frac{3x+1}{x-2}} \rightarrow y^2 = \frac{3x+1}{x-2} \rightarrow y^2(x-2) = 3x+1$$

$$x(y^2-3) = 2y^2+1 \rightarrow x = \frac{2y^2+1}{(y-2)(y+2)} \rightarrow [0, 1) \cup (2, +\infty)$$