

$$f(-n) - n f(n) = 2|n| + 1 \xrightarrow{x=n}$$

$$f(n) + n f(-n) = 2|n| + 1 \xrightarrow{\text{در دو طرف } -n \text{ ضرب کنیم}}$$

$$\begin{cases} -n f(-n) + n^2 f(n) = -2|n| + n \\ n f(-n) + f(n) = 2|n| + 1 \end{cases}$$

$$(n^2 + 1) f(n) = 2|n| + 1 - 2|n| + n$$

$$f(n) = \frac{2|n| + 1 - 2|n| + n}{n^2 + 1}$$

$$\begin{aligned} n > 0 & \rightarrow \frac{-2n^2 + n + 1}{n^2 + 1} \\ n < 0 & \rightarrow \frac{2n^2 - 3n + 1}{n^2 + 1} \end{aligned}$$

$$f(-n) + f(n) = n^2 - 2n + 4 \xrightarrow{n=0} \text{ و } f(0) = 4 \rightarrow f(0) = 4$$

$$f(n) = n^2 - 2n + 4$$

$$f(n^2) = (n^2)^2 - 2(n^2) + 4 = n^4 - 2n^2 + 4$$

$$y = \frac{n^2 + n + 1}{n - 1}$$

$$y_n - y = n^2 + n + 1$$

$$n^2 + (1 - y)n + y = 0$$

$$\Delta = (1 - y)^2 - 4(y + 1) = y^2 - 4y - 3$$

$$\Delta \geq 0 \rightarrow y^2 - 4y - 3 \geq 0 \rightarrow (y - 7)(y + 1) \geq 0 \rightarrow \frac{-1}{+} \frac{7}{-}$$

$$R = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف) } y = 2x^2 - 2x + 1 \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{-b}{2a} = \frac{1}{2} \\ y_{\min} = \frac{-1}{2} \end{cases} \rightarrow R_f = \left[\frac{-1}{2}, +\infty \right)$$

$$\text{ب) } y = \sqrt{-x^2 + 4x - 5} \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{\max} = 1 \end{cases} \rightarrow R_f = \sqrt{[-\infty, 4]} = [0, 2]$$

$$\text{ج) } \sqrt{\frac{n^2 - 2n + 1}{n}} \rightarrow R_f = \sqrt{R} = [0, +\infty)$$

$$\text{د) } y = \log\left(\frac{n^2 - 4n + 4}{n}\right) \rightarrow R_f = (-\infty, +\infty)$$

$$y = \frac{n+1}{n-4} \rightarrow R = \left\{ \frac{a}{c} \right\} = R_f = \mathbb{R} - \left\{ \frac{4}{1} \right\}$$

$$y = \sqrt{\frac{n+1}{n-4}} \rightarrow R_f = \sqrt{R - \{4\}} = [0, +\infty) - \{2\}$$

$$y = \frac{n^2 + 4}{\sqrt{n^2 + 4}} = \frac{n^2 + 4}{\sqrt{n^2 + 4}} + \frac{1}{\sqrt{n^2 + 4}} \xrightarrow{\sqrt{n^2 + 4} > 0} R = \left[\frac{4}{\sqrt{4}}, +\infty \right)$$

دوین کمترینه معادلی: $\sqrt{n^2 + 4}$ و $\frac{1}{\sqrt{n^2 + 4}}$ داشته باشند و هر دو مثبت است پس کمترینه معادله است: $\sqrt{n^2 + 4} + \frac{1}{\sqrt{n^2 + 4}} = \frac{4}{\sqrt{n^2 + 4}}$

$$y = n^2 + \frac{1}{n^2 + 4} = n^2 + 4 + \frac{1}{n^2 + 4} - 4 \xrightarrow{n^2 + 4 > 0} R = \left[\frac{1}{4}, +\infty \right)$$

کمترین مقدار: $n^2 + 4$ است پس کمترینه مقدار: $4 + \frac{1}{4} - 4 = \frac{1}{4}$

$$y = \frac{3x^2 + 4}{x^2 - 1}$$

برداشت اولی: $y = \frac{3x^2 + 4}{x^2 - 1} \rightarrow x^2(y - 3) = y + 4 \rightarrow x = \sqrt{\frac{y+4}{y-3}}$

$$\frac{y+4}{y-3} \geq 0 \rightarrow \frac{-4}{+6-6} \rightarrow R = (-\infty, -4] \cup (3, +\infty)$$

برداشت دومی: $y = \frac{3x^2 + 4}{x^2 - 1} \rightarrow \begin{matrix} x=+\infty & 3 \\ x=0 & -4 \end{matrix}$
 مقدار مخرج $x^2 - 1$
 $R = (-\infty, -4] \cup (3, +\infty)$
 چون $y = 3$ هرگز حاصل نمی شود
 و این مقدار رو به دست آوریم ✓

$$f(x) = \frac{4 \sin x + 4}{\sin x - 3}$$

$$\begin{matrix} x=1 & -3 \\ x=-1 & -\frac{1}{3} \end{matrix}$$

$$R = [-3, -\frac{1}{3}]$$

$$y = \left| \frac{3x-1}{x} \right|$$

چون قدر مطلق داریم $y \geq 0$ (II)

$$\frac{3x-1}{x} = y \rightarrow 3x-1 = xy \rightarrow x(3-y) = 1$$

$$x = \frac{1}{3-y} \rightarrow 3-y \neq 0 \rightarrow y \neq 3 \quad (I)$$

$$R = [0, +\infty)$$

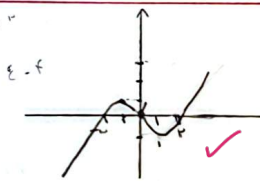
$$\frac{3x-1}{x} = -y \rightarrow 3x-1 = -xy \rightarrow x(3+y) = 1$$

$$x = \frac{1}{3+y} \rightarrow y \neq -3 \quad (II)$$

$$\sqrt{\frac{9x-1}{-x+3}} = \sqrt{\frac{9(x-1)}{x(3-x)}} = \sqrt{\frac{9}{x}} = \frac{3}{\sqrt{x}} \quad R = \left\{ \frac{3}{\sqrt{x}} \right\}$$

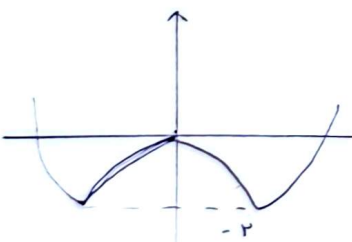
$$R = \left\{ \frac{3}{\sqrt{x}} \right\}$$

$$y = x|x| - 2x \rightarrow \begin{matrix} y = x^2 - 2x & x > 0 \\ y = -x^2 - 2x & x < 0 \end{matrix}$$

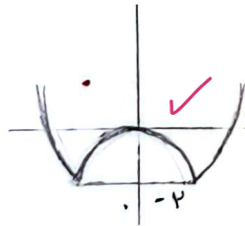


$$R = \mathbb{R}$$

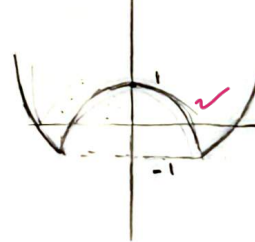
$$y = |f(x)| + 1$$



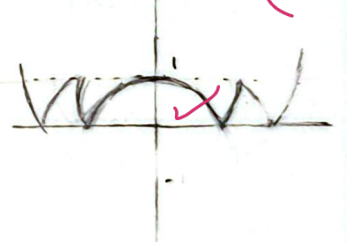
$$f(x)$$



$$f(x) + 1$$



$$|f(x)| + 1$$



$$R_y = |\mathbb{R} - \left\{ \frac{3}{4} \right\}| = [0, +\infty)$$

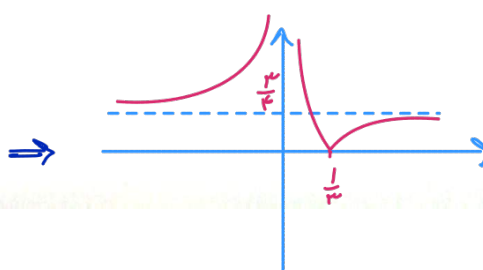
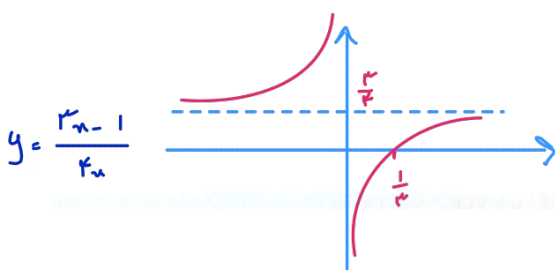
۸- اول برد عبارت داخل قدر مطلق

رو حساب می کنیم بعد قدر مطلق می گیریم

فقط حواسه باشه که $\frac{3}{4} = 0.75$ هم هست داخل برد

$y = \frac{3}{4}$ عضو برد عبارت داخل قدر مطلق هست و وقتی قدر مطلق بگیریم $y = \frac{3}{4}$

روید به نمودار



$$y = \left| \frac{3x-1}{x} \right|$$