

$$f(-x) - xf(x) = 2|x| + 1 \xrightarrow{x \rightarrow -x} \int x f(-x) + x^2 f(x) = -2|x| - 2x$$

$$(x \text{ مبداء}) f(x) + x f(-x) = 2|x| + 1 \quad \uparrow \quad f(x) + x f(-x) = 2|x| + 1$$

$$x^2 f(x) + f(x) = -2|x| + 2|x| - x + 1$$

$$f(x) = \frac{-2|x| + 2|x| - x + 1}{x^2 + 1}$$

$$f(x) = \begin{cases} \frac{-2x + 1}{x^2 + 1} & x \geq 0 \\ \frac{2x - 1}{x^2 + 1} & x < 0 \end{cases}$$

$$f(x) + f(0) = x^2 - 3x + 4 \xrightarrow{x=0} 2f(0) = 4 \quad f(0) = 2$$

$$f(x) = x^2 - 3x + 3$$

$$f(x^2) = (x^2)^2 - 3(x^2) + 3 \Rightarrow f(x^2) = x^4 - 3x^2 + 3$$

$$y = \frac{x^2 + x + 2}{x - 1} \quad yx - y = x^2 + x + 2 \quad x^2 + (1-y)x + y + 2 = 0$$

$$x = \frac{y-1 \pm \sqrt{(y-1)^2 - 4(y+2)}}{2} \quad (y-1)^2 - 4(y+2) \geq 0 \quad y^2 - 4y + 1 - 4y - 8 \geq 0$$

$$y^2 - 4y - 7 \geq 0 \quad (y-7)(y+1) \geq 0 \quad R_f = (-\infty, -1] \cup [7, +\infty)$$

بر روی منبسط‌کننده در مقدار منبسط‌کننده است

$$y = 2x^2 - 5x + 1 \quad \min \left| \frac{a}{x} \right| \quad R_{\text{اف}} = \left[-\frac{14}{x}, +\infty \right)$$

$$y = \sqrt{x^2 + 4x - 5} \quad \max |x| \quad R_{\text{ب}} = [0, 2]$$

$$y = \sqrt{x^2 - 12x + 20} \quad R_{\text{ع}} = \sqrt{12} = [0, +\infty)$$

$$y = \log(x^2 - 4x^2 + 12x + 4) \rightarrow \log 12 \rightarrow \log(+\infty) \Rightarrow R_{\text{د}} = (-\infty, +\infty) = \mathbb{R}$$

$$y = \frac{3x+1}{2x-4} \xrightarrow{\text{مبداء}} \frac{3x+1}{2x-4} = y \quad x = \frac{-4y-1}{2y-3} \quad x = \frac{4y-1}{2y-3} \quad y \neq \frac{3}{2} \quad R_f = \mathbb{R} - \left\{ \frac{3}{2} \right\}$$

$$y = \sqrt{\frac{x+1}{x-2}} \quad R_f = \sqrt{12 - \frac{1}{x}} = [0, +\infty) - \left\{ \frac{1}{2} \right\}$$

$$y = \frac{x^2 + 4}{\sqrt{x^2 + 4}} = \frac{x^2 + 4}{\sqrt{x^2 + 4}} + \frac{1}{\sqrt{x^2 + 4}} \rightarrow \min: \sqrt{\frac{4}{x^2 + 4}} + \frac{1}{\sqrt{\frac{4}{x^2 + 4}}} \left(\frac{4}{x^2 + 4} \right)$$

$$y = x^2 + \frac{1}{x+2} + 4 - 4 \quad R = \left[\frac{14}{x} - 4, +\infty \right) = \left[\frac{1}{x}, +\infty \right)$$

مقدار منبسط‌کننده (مبداء)

$$\min: 4 + \frac{1}{x} = \frac{14}{x}$$

$$y = \frac{px^2 + k}{x^2 - 1} \xrightarrow{\text{مربع 20}} yx^2 - y = px^2 + k \quad (p-y)x^2 + y + k = 0 \quad x = \frac{0 \pm \sqrt{-(p-y)(y+k)}}{p-y}$$

$$-k(p-y)(y+k) \geq 0 \quad + \quad - \quad - \quad + \quad R_y = (-\infty, -k] \cup (p, +\infty)$$

$$y \neq p$$

$$\xrightarrow{\text{مربع 20}} \min x^2 = 0 \rightarrow (p-y) = 0 \quad y = \frac{p+k}{p-1} = -k$$

$$\max x^2 = +\infty \rightarrow (p-y) = 0 \quad y = \frac{p+k}{p-1} = p$$

پیدا کردن دامنه

$$R_y = (-\infty, -k] \cup (p, +\infty)$$

$$f(x) = \frac{p \sin x + k}{\sin x - p}$$

خرج متناهی من زدن ندارد

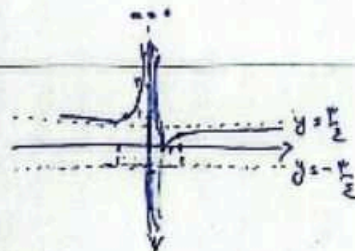
$$\min \sin x = -1 \rightarrow f(x) = \frac{-p+k}{-1-p} = \frac{1}{p}$$

$$R_f = \left[-\frac{p}{1-p}, \frac{1}{p}\right]$$

$$\max \sin x = 1 \rightarrow f(x) = \frac{p+k}{1-p} = -p$$

$$\text{الف) } y = \left| \frac{px-1}{x} \right| \xrightarrow{\text{مربع 20}} \frac{p^2-1}{x^2} + \frac{2p-1}{x} + \frac{1}{x^2}$$

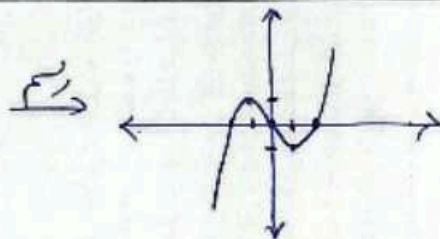
$$R_y = [0, +\infty)$$



$$\text{ب) } y = \sqrt{\frac{q(x-p)}{f(x-p)}} \rightarrow y = \sqrt{\frac{q}{f}} \quad x \neq p \quad y = \frac{p}{f}$$

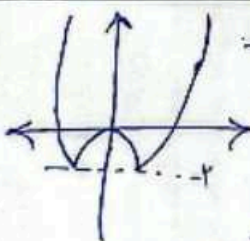
$$R_y = \left[\frac{p}{f}, +\infty\right)$$

$$y = \begin{cases} x^2 - 1 & x \geq 0 \\ -x^2 - 1 & x < 0 \end{cases}$$



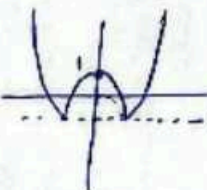
$$R_y = \mathbb{R}$$

$$\text{1) } f(x)$$



$$\text{2) } f(1/2) = f(x)$$

$$\text{3) } f(x) + 1$$



$$\text{4) } |f(x) + 1|$$

نیمه را با یک واحد در می بینیم

