

۲۰ آفرین:

$$1) f(-x) - x f(x) = |x| + 1 \xrightarrow{x=1} f(-1) - 1 f(1) = |1| + 1$$

$$f(1) + x f(x) = |1-x| + 1 \xrightarrow{x=1} f(1) + 1 f(1) = |1-1| + 1$$

(۲)

$$f(x) (x^2 - 1) = \frac{|x| + 1 + x - |x| - 1}{-x^2 - 1} \rightarrow f(x) = \frac{x|x| + x - |x| - 1}{-x^2 - 1}$$

$$\xrightarrow{x=1} \frac{1 \cdot 1 + 1 - 1 - 1}{-1 - 1} = \frac{0}{-2} = 0$$

$$\xrightarrow{x=-1} \frac{-1 \cdot 1 + (-1) - (-1) - 1}{-1 - 1} = \frac{-2}{-2} = 1$$

$$2) f(x) + f(0) = x^2 - 3x + 4 \xrightarrow{x=0} f(0) + f(0) = 0 - 0 + 4, 4 \rightarrow f(0) = 4$$

$$f(x) + 4 = x^2 - 3x + 4 \rightarrow f(x) = x^2 - 3x$$

(۲)

$$3) y = \frac{x^2 + x + 1}{x - 1} \rightarrow y(x-1) = x^2 + x + 1 \rightarrow x^2 + (1-y)x + 1 + y = 0$$

(۲)

$$x = \frac{y-1 \pm \sqrt{\Delta}}{1} \rightarrow \Delta = (y)^2 - 4(1+y) = y^2 - 4y - 4 = y^2 - 4y - 4$$

$$\Delta \geq 0 \rightarrow y^2 - 4y - 4 \geq 0 \rightarrow (y-2)(y+2) \geq 0 \rightarrow y \in (-\infty, -2] \cup [2, +\infty)$$

$$4) \text{ الف) } y = x^2 - 2x + 1 \text{ ext } \left| \frac{b}{a} \right| = \frac{1}{1} = 1$$

$$x = \frac{1 \pm \sqrt{1-4(1-1)}}{2} = \frac{1 \pm 0}{2} = \frac{1}{2}$$

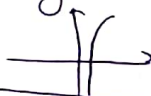
(۲)

$$ب) y = \sqrt{-x^2 + 4x - 5} \text{ ext } \left| \frac{b}{a} \right| = \frac{2}{1} = 2$$

$$-x^2 + 4x - 5 \geq 0 \rightarrow x \in [1, 5]$$

$$ج) y = \sqrt{x^2 - 2x + 1} \text{ بر مبنای } x \text{ در } [0, +\infty) \text{ و } (-\infty, +\infty)$$

$$د) y = \log(x^2 - 2x + 1) \text{ بر مبنای } x \text{ در } (0, +\infty) \text{ و } (-\infty, +\infty)$$



$$5) y = \frac{x+1}{x-1} \text{ بر مبنای } x \text{ در } \mathbb{R} \setminus \{1\}$$

(۲)

$$ب) y = \sqrt{\frac{x+1}{x-1}} \text{ بر مبنای } x \text{ در } (-\infty, +\infty) \setminus \{1\}$$

$$ج) y = \frac{x^2 + 1}{\sqrt{x^2 + 1}} = \sqrt{x^2 + 1} + \frac{1}{\sqrt{x^2 + 1}} \rightarrow \min y = \sqrt{1} + \frac{1}{\sqrt{1}} = 2 \rightarrow y \in [2, +\infty)$$

$$د) y = x^2 + \frac{1}{x^2 + 1} \rightarrow x^2 + 1 + \frac{1}{x^2 + 1} - 1 \rightarrow \min y = 1 + \frac{1}{1} = 2 \rightarrow y \in [2, +\infty)$$

$$y_{2t} - y_{1t} + K \rightarrow n^r(y-r), y+K \rightarrow \frac{y+K}{y-r} \cdot n^r \quad (2)$$

$\frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$
 Partial Fraction Decomposition:
 $\frac{1}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2}$
 $1 = A(s+1) + B$
 $1 = As + A + B$
 $0s + 1 = As + (A+B)$
 $A = 0$
 $A+B = 1 \Rightarrow 0+B=1 \Rightarrow B=1$
 $\frac{1}{(s+1)^2} = \frac{0}{s+1} + \frac{1}{(s+1)^2} = \frac{1}{(s+1)^2}$
 Laplace Transform Table:
 $\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} = t e^{-t}$

V) $\frac{r \sin x + r}{\sin x - r}$ $y \sin x - ry, r \sin x + r \rightarrow \sin x(y-r) + ry + r$

$\sin x \rightarrow \frac{ry + r}{y-r} \rightarrow 1 \geq \frac{ry + r}{y-r} \geq -1$

$\Rightarrow \frac{ry + r - y + r}{y-r} \rightarrow \frac{ry + r}{y-r} \rightarrow \frac{-r}{1-r-r} \rightarrow (E, r)$

$\frac{ry + r - y - r}{y-r} \geq 0 \rightarrow \frac{ry + r}{y-r} \geq 0 \rightarrow \frac{-1}{1-r-r} \rightarrow (-\frac{1}{r})$

$\} \rightarrow [-r, \frac{1}{r}]$

$U(r, +\infty)$

$\Delta) \lim_{x \rightarrow \infty} \left| \frac{f_n - 1}{f_n} \right|$
 $x \in [0, +\infty)$

$$b) y = \sqrt{\frac{9n-11}{-1+5n}} = \sqrt{\frac{9(\cancel{n}-1)}{1+5(\cancel{n}-1)}} = \frac{3}{5} \therefore R = \left\{ \frac{3}{5} \right\}$$

9) $y \sin x - y_n$ $\frac{0}{-x^2 - y_n \mid x^2 + y_n}$  (r)

