

1

$$f(-n) - n f(n) = 2|n| + 1 \xrightarrow{x=n} -x f(n) + n f(-n) = n(2|n| + 1) \quad (1)$$

$$f(n) + n f(-n) = 2|n| + 1 \quad (2)$$

از (1) و (2) می‌توانیم بنویسیم:

$$-(n^2 + 1) f(n) = (n-1)(2|n| + 1) \rightarrow f(n) = \frac{(1-n)(2|n| + 1)}{n^2 + 1}$$

2

$$f(n) + f(0) = n^2 - 3n + 4 \xrightarrow{n=0} 2f(0) = 4 \rightarrow f(0) = 2$$

$$\rightarrow f(n) = n^2 - 3n + 3 \quad \text{برای } n^2 \text{ و } n \text{ و } 3 \text{ بررسی می‌کنیم} \rightarrow f(n^2) = n^4 - 3n^2 + 3$$

3

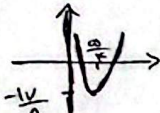
$$y = \frac{n^2 + n + 2}{n-1} \rightarrow n^2 + n + 2 = y(n-1) \rightarrow n^2 + (1-y)n + (2-y) = 0$$

$$\Delta = (1-y)^2 - 4(2-y) = y^2 - 4y - 7$$

$$\Delta \geq 0 \rightarrow y^2 - 4y - 7 \geq 0 \rightarrow \frac{-1}{+} \frac{4}{-} \frac{-7}{+} \rightarrow R_f = \mathbb{R} - (-1, 7)$$

4

$$الف) y = 2n^2 - 5n + 1 \quad \min \left| -\frac{b}{2a} = \frac{5}{4} \right|$$

$$2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) + 1 = -\frac{17}{8}$$


$$R_f = \left[-\frac{17}{8}, +\infty\right)$$

ب) $y = \sqrt{-n^2 + 4n} - 5$

$$\max \left| -\frac{b}{2a} = 2 \right| \rightarrow \sqrt{(-\infty, 4]} \rightarrow R_f = [0, 2]$$

ج) $y = \sqrt{n^4 - 2n^2 + 4n + 1}$

بر عبارت زیر رادیکال بررسی می‌کنیم $\rightarrow \sqrt{\mathbb{R}} \rightarrow R_f = [0, +\infty)$

د) $y = \log(n^2 - 4n^2 + 5n + 4)$

بر عبارت زیر لگاریتم بررسی می‌کنیم $\rightarrow \log(\mathbb{R}) \rightarrow R_f = \mathbb{R}$

5

$$الف) y = \frac{3n+1}{n-4} \rightarrow R_f = \mathbb{R} - \left\{\frac{4}{3}\right\} \rightarrow R_f = \mathbb{R} - \left\{\frac{4}{3}\right\}$$

ب) $y = \sqrt{\frac{4n+1}{n-2}}$

بر عبارت زیر رادیکال $= \mathbb{R} - \{2\} \rightarrow R_f = \sqrt{\mathbb{R} - \{2\}} \rightarrow R_f = [0, +\infty) - \{2\}$

ج) $y = \frac{n^2 + 4}{\sqrt{n^2 + 3}} = \frac{n^2 + 3 + 1}{\sqrt{n^2 + 3}} = \frac{\sqrt{n^2 + 3}}{1} + \frac{1}{\sqrt{n^2 + 3}}$

برای $n \rightarrow \infty$ $\sqrt{n^2 + 3} \rightarrow \sqrt{3}$

$$R_f = \left[\sqrt{3} + \frac{1}{\sqrt{3}}, +\infty\right)$$

د) $y = n^2 + \frac{1}{n^2 + 4} + 4 - 4 = (n^2 + 4) + \frac{1}{(n^2 + 4)} - 4$

$$R_f = \left[\frac{1}{4}, +\infty\right)$$

6

$$y = \frac{3n^2 + 4}{n^2 - 1}$$

برای n^2 و 4 و -1 بررسی می‌کنیم:

$$\frac{3n^2 + 4}{n^2 - 1} = y \rightarrow n^2 = -\frac{y-4}{y-3} \rightarrow \frac{y-4}{y-3} \geq 0$$

$$\frac{y-4}{y-3} \geq 0 \rightarrow R_f = \mathbb{R} - (-4, 3]$$

بررسی می‌کنیم:

$$n^2 = 0 \rightarrow y = \frac{4}{-1} = -4$$

$$n^2 = +\infty \rightarrow y = 3$$

چون بین منفی و مثبت می‌شود $\rightarrow R_f = (-\infty, -4] \cup (3, +\infty)$

$$f(m) = \frac{r \sin m + r}{\sin m - r}$$

نکته: ⑤

$$\sin m = 1 \rightarrow f(m) = \frac{r}{-r} = -1$$

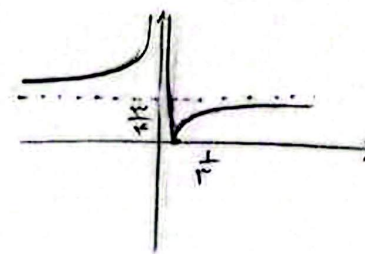
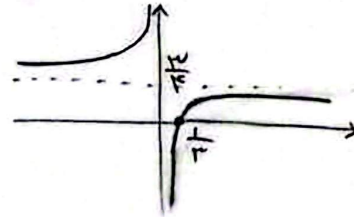
$$\sin m = -1 \rightarrow f(m) = \frac{r}{-r} = -1$$

چون مخرج صفر نمی شود

$$\rightarrow R_f = \left[-1, -\frac{1}{r}\right]$$

⑤

$$\text{الف) } y = \left| \frac{r m - 1}{r m} \right|$$

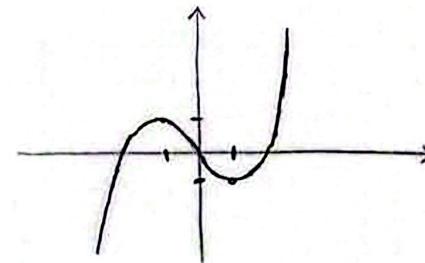


$$R_f = [0, +\infty)$$

①

$$\text{ب) } y = \sqrt{\frac{r m - 1}{r m - 1}} = \sqrt{\frac{\frac{r}{r}(r m - 1)}{(r m - 1)}} = \sqrt{\frac{r}{r}} = \frac{r}{r} \rightarrow R_f = \left\{ \frac{r}{r} \right\}$$

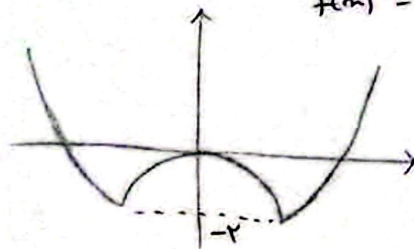
$$y = x|x| - r x \rightarrow y = \begin{cases} x^2 - r x & ; x \geq 0 \\ -x^2 - r x & ; x \leq 0 \end{cases}$$



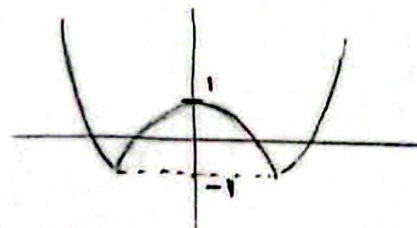
$$R_f = \mathbb{R}$$

⑨

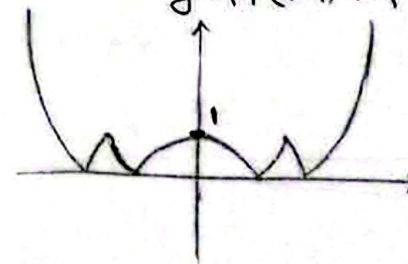
$$f(m) = f(|m|)$$



$$f(m) + 1$$



$$y = |f(m) + 1|$$



⑩