

$$\begin{aligned} \begin{cases} f_{(-n)} - n f_{(n)} = 2|n| + 1 \\ f_{(n)} + n f_{(-n)} = 2|-n| + 1 \end{cases} & \Rightarrow \begin{cases} -n f_{(-n)} + n^2 f_{(n)} = -2n|n| - n \\ n f_{(-n)} + f_{(n)} = 2|-n| + 1 \end{cases} \\ n^2 f_{(n)} + f_{(n)} = -2n|n| + 2|-n| + 1 - n & \\ (n^2 + 1) f_{(n)} = -2n|n| + 2|-n| - n + 1 & \end{aligned}$$

$$f_{(n)} = \begin{cases} \frac{-2n^2 + n + 1}{n^2 + 1} & n \geq 0 \\ \frac{2n^2 - 2n + 1}{n^2 + 1} & n < 0 \end{cases}$$

1

$$f_{(n)} + f_{(0)} = n^2 - 2n + 4 \rightarrow n = 0 \Rightarrow 2f_{(0)} = 4 \rightarrow f_{(0)} = 2$$

$$f_{(n)} = n^2 - 2n + 3 \rightarrow n = n^2 \Rightarrow f_{(-n^2)} = n^4 - 2n^2 + 3$$

2

$$y = \frac{n^2 + n + 2}{n - 1} \rightarrow y(n - 1) = n^2 + n + 2 \Rightarrow n^2 + (-y + 1)n + y + 2 = 0$$

$$n = \frac{y - 1 \pm \sqrt{y^2 - 4y + 1 - 4y + 8}}{2} \Delta \geq 0 \Rightarrow y^2 - 4y - 7 \geq 0$$

$$\frac{-1}{+} \frac{7}{-}$$

3

$$R_f = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف) } y = \frac{n^2 - 4n + 1}{n - 2} \quad 2 \times \frac{16}{14} - \frac{16}{2} + 1 = -\frac{10}{7}$$

$$n_2: -\frac{b}{2a} = \frac{2}{2} \quad R_y = [-\frac{10}{7}, +\infty)$$

$$\text{ب) } y = \sqrt{-n^2 + 4n - 4}$$

$$\text{max} = \sqrt{1 - 4 + 4 - 4} = 2 \quad R_y = [0, 2]$$

4

$$\text{ج) } y = \sqrt{n^2 - 4n^2 + 4n + 1}$$

$$\text{د) } y = \sqrt{R} = [0, +\infty)$$

$$\text{د) } y = \log(n^2 - 4n^2 + 4n + 1) \rightarrow$$

$$R_y = \mathbb{R}$$

$$\text{الف) } y = \frac{n^2 + 1}{n - 2}$$

$$R_y = \mathbb{R} - \left\{ \frac{5}{2} \right\} \rightarrow \mathbb{R} - \left\{ \frac{5}{2} \right\}$$

$$\text{ب) } y = \sqrt{\frac{n^2 + 1}{n - 2}} \Rightarrow \sqrt{R - \frac{5}{2}}$$

$$R_y = [0, +\infty) \setminus \left\{ \frac{5}{2} \right\}$$

5

$$\text{ج) } y = \frac{n^2 + 2}{\sqrt{n^2 + 2}} = \sqrt{n^2 + 2} + \frac{1}{\sqrt{n^2 + 2}}$$

$$\text{min} = \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{3\sqrt{2}}{2} \quad R_y = \left[\frac{3\sqrt{2}}{2}, +\infty \right)$$

$$\text{د) } y = n^2 + \frac{1}{n^2 + 2} \rightarrow \text{min, max}$$

$$0 + \frac{1}{2} = \frac{1}{2} \quad R_y = \left[\frac{1}{2}, +\infty \right)$$

$$y = \frac{r_n r + \varepsilon}{n^r - 1} \Rightarrow y n^r - y = r_n r + \varepsilon \Rightarrow (y - r) n^r = y + \varepsilon \rightarrow n^r = \frac{y + \varepsilon}{y - r}$$

$$|n| = \pm \sqrt{\frac{y + \varepsilon}{y - r}} \quad \frac{y + \varepsilon}{y - r} \rightarrow \frac{-\varepsilon}{r - \frac{1}{y} + r} \quad R_y = (-\infty, -\varepsilon] \cup (r, +\infty)$$

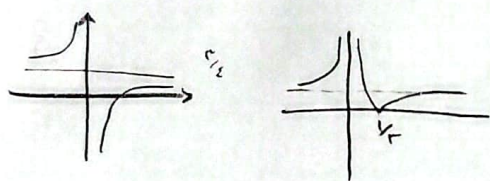
6

$$y = \frac{r_n r + \varepsilon}{n^r - 1} \quad \begin{array}{l} n=0 \quad y = -\varepsilon \\ n=+\infty \quad y = r \end{array} \quad R_y = (-\infty, -\varepsilon] \cup (r, +\infty)$$

$$f(n) = \frac{r \sin n + \varepsilon}{\sin n - r} \quad \begin{array}{l} \sin n = 1 \Rightarrow \frac{r}{-r} = -1 \\ \sin n = -1 \Rightarrow \frac{r}{-r} = -1 \\ \frac{c}{a} = r \quad \times \end{array} \quad R_y = [-1, -\frac{1}{r}]$$

7

$$a) y = \left| \frac{r_n - 1}{r_n} \right| \quad R_y = [0, +\infty) \quad b) y = \sqrt{\frac{q_n - 1}{-n + \varepsilon n}} = \sqrt{\frac{q(n^r)}{r(n - r)}} = \frac{r}{r}$$

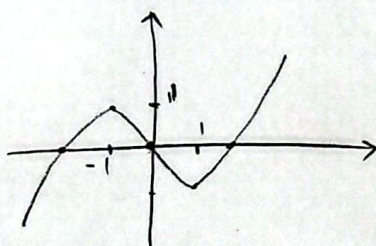


$$R_y = \left\{ \frac{r}{r} \right\}$$

8

$$y = 2|n| - r_n$$

$$y = \begin{cases} n^r - r_n & n > 0 \\ 0 & n = 0 \\ -n^r - r_n & n < 0 \end{cases}$$

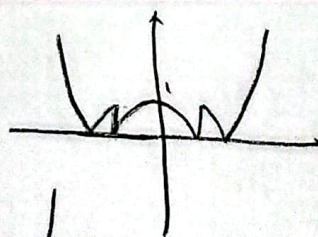


$$R_y = \mathbb{R}$$

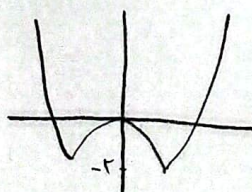
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$$y = |f(n) + 1|$$

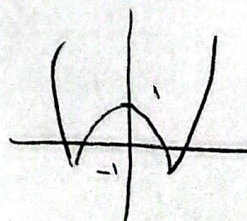
$$y = |f(n) + 1| \Rightarrow$$



$$y = f(n)$$



$$y = f(n) + 1$$



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