

19, 20

Colours 16

(1)

$$f(x) + x f(-x) = 2|x| + 1$$

$$f(-x) - x f(x) = -2|x| - x \cdot x \rightarrow -x f(-x) + x^2 f(x) = -2|x| - x$$

$$x^2 f(x) + f(x) = -2|x| - x + 2|x| + 1$$

$$f(x)(x^2 + 1) = -2|x| - x + 2|x| + 1 \rightarrow f(x) = \frac{-2|x| - x + 2|x| + 1}{x^2 + 1}$$

$$x > 0 \rightarrow f(x) = \frac{-2x + 1 - 2x^2 - x}{x^2 + 1} = \frac{-2x^2 - 3x + 1}{x^2 + 1}$$

$$x < 0 \rightarrow f(x) = \frac{2x^2 - x - 2x + 1}{x^2 + 1} = \frac{2x^2 - 3x + 1}{x^2 + 1}$$

$$f(x) + f(0) = -2x^2 - 3x + 4 \xrightarrow{x=0} f(0) + f(0) = 4 \rightarrow f(0) = 2$$

$$f(x) + 2 = 2x^2 - 3x + 4 \rightarrow f(x) = 2x^2 - 3x + 2 \rightarrow f(x^2) = 2x^4 - 3x^2 + 2$$

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow y(x - 1) = x^2 + x + 2 \rightarrow x^2 + (1 - y)x + (y + 2) = 0 \rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow \Delta \geq 0$$

$$(1 - y)^2 - 4(y + 2) \geq 0 \rightarrow y^2 - 2y + 1 - 4y - 8 \geq 0 \rightarrow y^2 - 6y - 7 \geq 0$$

$$\frac{-y}{+} \quad \frac{y}{-}$$

$$\rightarrow R_y = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف) } y = 2x^2 - 3x + 1 \quad \left| \begin{array}{l} x_5 = -\frac{b}{2a} = \frac{3}{4} \\ y_5 = -\frac{W}{4a} \end{array} \right. \xrightarrow{\Delta \geq 0} R_y = \left[ -\frac{W}{4} + \infty \right)$$

$$\rightarrow y = \sqrt{-x^2 + 4x - 2} \quad \left| \begin{array}{l} x_5 = +2 \\ y_5 = 2 \end{array} \right. \xrightarrow{\Delta \geq 0} \sqrt{(-x_5)^2 - 4} \rightarrow [0, 4]$$

$$\text{ب) } y = \sqrt{x^2 - 4x^2 + 4x + 1} \rightarrow \text{نقطة الصفر} \rightarrow \sqrt{R} \rightarrow [0, 1]$$

$$\text{ج) } y = \log(x^2 - 4x^2 + 4x + 1) \rightarrow \text{نقطة الصفر} \rightarrow \log(0) \rightarrow R_y \leq R$$

$$\text{د) } y = \frac{(x^2 + 1)}{(x^2 - 4)} \rightarrow R_y = R - \{ \frac{1}{4} \}$$

$$\rightarrow y = \sqrt{\frac{x^2 + 1}{x^2 - 4}} \rightarrow R - \{ \frac{1}{2} \} \rightarrow R_y = [0, +\infty) - \{ \frac{1}{2} \}$$

$$\text{هـ) } y = \frac{x^2 + 1}{\sqrt{x^2 + 4}} = \sqrt{x^2 + 4} + \frac{1}{\sqrt{x^2 + 4}} \rightarrow R_y = \left[ \frac{1}{\sqrt{4}} + \infty \right)$$

$$\text{و) } y = \frac{x^2 + 1}{x^2 + 4} \rightarrow \min = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \rightarrow R_y = \left[ \frac{1}{2}, +\infty \right)$$

(2) 16

$$y = \frac{3x^2 + \varepsilon}{x^2 - 1} \rightarrow x^2 = -\frac{y - \varepsilon}{y - 3} = \frac{y + \varepsilon}{y - 3} \rightarrow x = \pm \sqrt{\frac{y + \varepsilon}{y - 3}}$$

$$\frac{-\varepsilon}{3} \leq \frac{y + \varepsilon}{y - 3} \rightarrow R_y = (-\infty, -\varepsilon] \cup (3, +\infty)$$

$$x^2 = 0 \rightarrow y = \frac{\varepsilon}{-1} = -\varepsilon$$

$$x^2 = +\infty \rightarrow y = 3$$

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(2) 17

$$f(x) = \frac{3 \sin x + \varepsilon}{\sin x - 3}$$

$$\sin x = +1 \rightarrow \frac{3 + \varepsilon}{1 - 3} = -3$$

$$\sin x = -1 \rightarrow \frac{-3 + \varepsilon}{-1 - 3} = -\frac{1}{4}$$

$$R_y = [-\frac{1}{4}, -3]$$

(2) 18

$$y = \left| \frac{3x - 1}{\varepsilon x} \right| \rightarrow \frac{3x - 1}{\varepsilon x} = 0 \rightarrow 3x - 1 = 0 \rightarrow x = \frac{1}{3}$$

$$R_y = [0, +\infty)$$

$$\sqrt{\frac{9x - 1}{-1 + \varepsilon x}} = \sqrt{\frac{9(x - \frac{1}{3})}{\varepsilon(x - \frac{1}{3})}} \Rightarrow y = \sqrt{\frac{9}{\varepsilon}} = \frac{3}{\sqrt{\varepsilon}} \rightarrow R_y = \{\frac{3}{\sqrt{\varepsilon}}\}$$

(2) 19

$$y = a|x| - 2x$$

|                            |                          |
|----------------------------|--------------------------|
| $-x^2 - 2x$<br>$x(-x - 2)$ | $x^2 - 2x$<br>$x(x - 2)$ |
|----------------------------|--------------------------|

$$R_y = \mathbb{R}$$

$$y = |f(|x|) + 1|$$

$$R_y = [0, +\infty)$$

۸- اول برد عبارت داخل قدر مطلق  
رد حساب می کنیم بعد قدر مطلق می گیریم

فقط خواست باشد که  $y = \frac{3}{4}$  و هم هست داخل برد

$y = -\frac{3}{4}$  عضو برد عبارت داخل قدر مطلق هست و وقتی قدر مطلق بگیریم  $y = \frac{3}{4}$

روید به نمودار:

