

(1)

$$f(x) + x f(-x) = 2|x| + 1$$

$$f(-x) - x f(x) = -2|x| - x \cdot x \rightarrow -x f(-x) + x^2 f(x) = -2|x| - x$$

$$x^2 f(x) + f(x) = -2|x| - x + 2|x| + 1$$

$$f(x)(x^2 + 1) = -2|x| - x + 2|x| + 1 \rightarrow f(x) = \frac{-2|x| - x + 2|x| + 1}{x^2 + 1}$$

$$x > 0 \rightarrow f(x) = \frac{-2x + 1 - 2x^2}{x^2 + 1} = \frac{-2x^2 - 2x + 1}{x^2 + 1}$$

$$x < 0 \rightarrow f(x) = \frac{2x - 1 - 2x^2}{x^2 + 1} = \frac{-2x^2 + 2x - 1}{x^2 + 1}$$

$$f(x) + f(0) = -2x^2 - 2x + 1 \xrightarrow{x=0} f(0) + f(0) = 1 \rightarrow f(0) = \frac{1}{2}$$

$$f(x) + \frac{1}{2} = -2x^2 - 2x + \frac{3}{2} \rightarrow f(x) = -2x^2 - 2x + \frac{3}{2} \rightarrow f(x^2) = -2x^4 - 2x^2 + \frac{3}{2}$$

$$y = \frac{x^2 + x + 1}{x - 1} \rightarrow y(x - 1) = x^2 + x + 1 \rightarrow x^2 + (1 - y)x + (1 + y) = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow x \geq 0$$

$$(1 - y)^2 - 4(1 + y) \geq 0 \rightarrow y^2 - 2y + 1 - 4 - 4y \geq 0 \rightarrow y^2 - 6y - 3 \geq 0$$

$$\frac{-y}{+} \quad \frac{y}{-}$$

$$\rightarrow R_y = (-\infty, -1] \cup [3, +\infty)$$

$$\text{الف) } y = \frac{x^2}{x^2 + 1} \quad \left| \begin{array}{l} x_5 = -\frac{b}{a} = 0 \\ y_5 = -\frac{1}{1} \end{array} \right. \xrightarrow{a > 0} R_y = [-1, 1]$$

$$\rightarrow y = \sqrt{-x^2 + 4x - 2} \quad \left| \begin{array}{l} x_5 = +2 \\ y_5 = 2 \end{array} \right. \xrightarrow{a < 0} \sqrt{(-2, 2]} \rightarrow [0, 2]$$

$$\text{ب) } y = \sqrt{x^2 - 4x^2 + 4x + 1} \rightarrow \text{نقطة الصفر} \rightarrow \sqrt{R} \rightarrow [0, +\infty)$$

$$\text{ج) } y = \log(x^2 - 4x^2 + 4x + 1) \rightarrow \text{نقطة الصفر} \rightarrow \log(0, +\infty) \rightarrow R_y = R$$

$$\text{د) } y = \frac{(x^2 + 1)}{(x^2 - 4)} \rightarrow R_y = R - \left\{ \frac{1}{4} \right\}$$

$$\rightarrow y = \sqrt{\frac{x^2 + 1}{x^2 - 4}} \rightarrow \sqrt{R - \{4\}} \rightarrow R_y = [0, +\infty) - \{2\}$$

$$\text{هـ) } y = \frac{x^2 + 1}{\sqrt{x^2 + 4}} = \sqrt{x^2 + 4} + \frac{1}{\sqrt{x^2 + 4}} \rightarrow R_y = \left[\frac{1}{\sqrt{4}} + \infty \right)$$

$$\text{و) } y = \frac{x^2 + 1}{x^2 + 4} \rightarrow \min = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \rightarrow R_y = \left[\frac{1}{2}, +\infty \right)$$

$$y = \frac{3x^2 + \varepsilon}{x^2 - 1} \rightarrow x^2 = -\frac{y - \varepsilon}{y + 3} = \frac{y + \varepsilon}{y - 3} \rightarrow x = \pm \sqrt{\frac{y + \varepsilon}{y - 3}}$$

$$\frac{-\varepsilon}{3} < \frac{y + \varepsilon}{y - 3} < \frac{-\varepsilon}{3} \rightarrow R_y = (-\infty, -\varepsilon] \cup (\varepsilon, +\infty)$$

$$x^2 = 0 \rightarrow y = \frac{\varepsilon}{-1} = -\varepsilon$$

$$x^2 = +\infty \rightarrow y = 3$$

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$$f(x) = \frac{3 \sin x + \varepsilon}{\sin x - 3}$$

$$\rightarrow \begin{cases} \sin x = +1 \rightarrow \frac{3 + \varepsilon}{1 - 3} = -3 \\ \sin x = -1 \rightarrow \frac{-3 + \varepsilon}{-1 - 3} = -\frac{1}{4} \end{cases} \rightarrow R_y = [-3, -\frac{1}{4}]$$

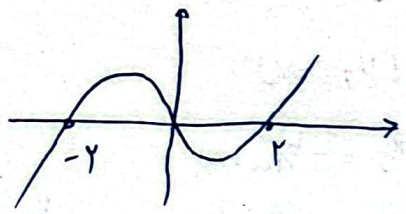
$$y = \left| \frac{3x - 1}{\varepsilon x} \right| \rightarrow \frac{3x - 1}{\varepsilon x} = 0 \rightarrow 3x - 1 = 0 \rightarrow x = \frac{1}{3} \checkmark$$

$$R_y = [0, +\infty)$$

$$\rightarrow \sqrt{\frac{9x - 11}{-1 + \varepsilon x}} = \sqrt{\frac{9(x - \frac{11}{9})}{\varepsilon(x - \frac{11}{9})}} \Rightarrow y = \sqrt{\frac{9}{\varepsilon}} = \frac{3}{\sqrt{\varepsilon}} \rightarrow R_y = \left\{ \frac{3}{\sqrt{\varepsilon}} \right\}$$

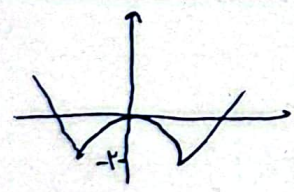
$$y = a|x| - 2x$$

$-x^2 - 2x$	$x^2 - 2x$
$x(-x - 2)$	$x(x - 2)$



$$R_y = \mathbb{R}$$

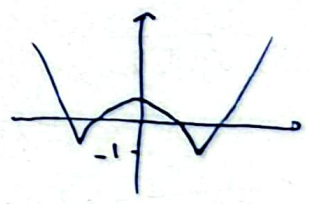
$$y = |f(|a|) + 1|$$



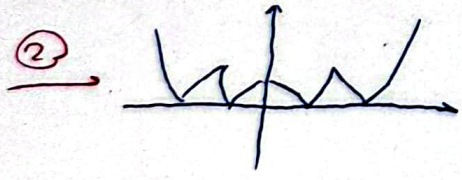
(1)



+1



(10)



(2)

$$R_y = [0, +\infty)$$