

$$\begin{aligned}
 & q_1 \rightarrow \left\{ \begin{aligned} (f(-q_1) - q_1 f(q_1) = 2|q_1| + 1) x - q_1 \\ f(q_1) + q_1 f(-q_1) = 2|-q_1| + 1 \end{aligned} \right. \quad f(q) = \frac{(2|q_1| + 1)(1 - q_1)}{q_1^2 + 1} \\
 & -q_1 \rightarrow \left\{ \begin{aligned} (f(q_1) - q_1 f(-q_1) = 2|q_1| + 1) x - q_1 \\ f(q_1) + q_1 f(-q_1) = 2|-q_1| + 1 \end{aligned} \right. \\
 & \frac{q_1^2 f(q_1) + f(q_1)}{f(q_1)(q_1^2 + 1)} = \frac{-2q_1|q_1| - q_1 + 2|-q_1| + 1}{2|q_1|(1 - q_1) + (1 - q_1)} \Rightarrow
 \end{aligned}$$

$$q_1 = 0 \Rightarrow \begin{aligned} 2f(0) = 4 \\ f(0) = 2 \end{aligned} \rightarrow f(q_1) = \frac{q_1^2 - 3q_1 + 4}{(q_1 - 2)(q_1 - 1) + 1} \Rightarrow f(q_1^2) = (q_1^2 - 2)(q_1^2 - 1) + 1$$

$$\begin{aligned}
 q_1^2 + q_1 + 2 &= 4q_1 - 4 \Rightarrow q_1^2 + (1 - 4)q_1 + 2 + 4 = 0 \\
 \Delta &= (1 - 4)^2 - 4 \cdot 2 - 1 = 0 \\
 q_1 &= \frac{-(1 - 4) \pm \sqrt{(1 - 4)^2 - 4 \cdot 2 - 1}}{2} \Rightarrow \frac{(1 - 4)^2 - 4 \cdot 2 - 1 \geq 0}{1 + 4^2 - 2 \cdot 4} \Rightarrow 4^2 - 4 \cdot 4 - 1 \geq 0 \\
 & \quad (4 - 4)(4 + 1) \geq 0 \Rightarrow \begin{array}{c} -1 \quad 4 + \infty \\ + \quad 0 \quad - \quad 0 \quad + \end{array} \\
 & \quad R = (-\infty, -1] \cup [4, +\infty)
 \end{aligned}$$

$$\begin{aligned}
 \text{الف) } \min \rightarrow \text{الترجم} \quad & \left| -\frac{b}{2a} = \frac{5}{4} \right. \\
 & \left. 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) + 1 = -\frac{11}{8} \Rightarrow R = \left[-\frac{11}{8}, +\infty\right)
 \end{aligned}$$

$$\begin{aligned}
 \text{ب) } \max \rightarrow \text{الترجم} \quad & \left| -\frac{b}{2a} = 3 \right. \\
 & \left. -9 + 11 - 5 = 3 \rightarrow (-\infty, 3] \xrightarrow{\text{جزء}} R = [0, 3]
 \end{aligned}$$

$$\begin{aligned}
 \text{ج) } R &= [0, +\infty) \rightarrow \text{چون جواب رادیکال فزونی} \\
 & \quad \text{زوجی حتماً صفراً یا مثبت است} \quad \text{د) } (-\infty, +\infty) = R
 \end{aligned}$$

$$\frac{3q_1 + 1}{2q_1 - 6} = 4 \rightarrow q_1 = -\frac{-4q_1 - 1}{2q_1 - 6} = \frac{4q_1 + 1}{2q_1 - 6} \Rightarrow R = \left\{\frac{1}{2}\right\}$$

$$\begin{aligned}
 \text{ب) } \text{تابع مربعی با رادیکال است} \Rightarrow R = \{4\} \xrightarrow{\text{جزء}} [0, +\infty) - \{2\} = R
 \end{aligned}$$

$$\begin{aligned}
 \text{ج) } \text{کمترین مقدار} = q_1 = 0 \Rightarrow R &= \left[\frac{4\sqrt{3}}{3}, +\infty\right) \\
 \text{چون مقدار مربع و جذبه دارد مثبت است} & \quad \text{د) } \text{کمترین مقدار} = q_1 = 0 \rightarrow R = \left[\frac{1}{4}, +\infty\right) \\
 \text{مقدار مربع و جذبه دارد مثبت است} & \quad \text{است}
 \end{aligned}$$

روش اولی: $y q_1^2 - y = 3q_1^2 + 4 \Rightarrow q_1^2(1-y) + 4 + y = 0 \Rightarrow q_1 = \frac{\pm \sqrt{-4(1-y)(4+y)}}{2(1-y)}$

$-4(1-y)(4+y) \geq 0 \Rightarrow 4+y-12 \geq 0 \Rightarrow -8 \leq y \leq 12$

روش دیگری: $y = \frac{3q_1^2 + 4}{q_1^2 - 1}$ $\begin{cases} q_1^2 = +\infty \rightarrow \frac{3(+\infty)^2 + 4}{(+\infty)^2} = 3 + \frac{4}{(+\infty)^2} = 3 \\ q_1^2 = 0 \rightarrow -4 \end{cases}$

$R = (-\infty, -4] \cup [3, +\infty)$

$\xrightarrow{\text{مخرج درازای 1 است (که این دو است) منفرد شود}} R = (-\infty, -4] \cup [3, +\infty)$

$f(q_1) = \frac{1 \sin q_1 + 4}{\sin q_1 - 1}$ $\begin{cases} \sin q_1 = 1 \rightarrow -3 \\ \sin q_1 = -1 \rightarrow -\frac{1}{2} \end{cases}$

$\xrightarrow{\text{مخرج درازای 3 است (منفرد شود که امکان پذیر نیست)}} [-3, -\frac{1}{2}] = R$

$|R - \{\frac{a}{c}\}| = R \Rightarrow |R - \{\frac{3}{4}\}| \xrightarrow{\text{در حلقه}} [0, +\infty) - \{\frac{3}{4}\}$

$\Rightarrow R = |R - \{\frac{a}{c}\}| \Rightarrow |R - \{\frac{3}{4}\}| \xrightarrow{\text{جذر}} [0, +\infty) - \{\frac{3}{4}\}$

$q_1 > 0 \rightarrow q_1^2 - 2\alpha \xrightarrow{\text{min, الاستریم}} \begin{cases} -\frac{b}{2a} = 1 \\ -1 \end{cases} \Rightarrow R = [-1, +\infty)$

$q_1 < 0 \rightarrow -q_1^2 - 2\alpha \xrightarrow{\text{max, الاستریم}} \begin{cases} -1 \\ 1 \end{cases} \Rightarrow R = (-\infty, 1]$

$\cup \rightarrow R_{\text{کل}} = (-\infty, 1] \cup [-1, +\infty)$

