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$$x \Rightarrow -x \Rightarrow f(x) + x f(-x) = 2|x| + 1$$

$$f(-x) - x f(x) = 2|x| + 1 \xrightarrow{x(-x)} \begin{cases} x f(-x) + f(x) = 2|x| + 1 \\ -x f(-x) + x^2 f(x) = (-2|x| - 1)x \end{cases}$$

$$(x^2 + 1)f(x) = 2(1-x)|x| + 1 - x$$

$$\Rightarrow f(x) = \frac{2(1-x)|x| + 1 - x}{x^2 + 1}$$

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$$x=0 \Rightarrow f(0) + f(0) = 0 - 2(0) + 1 \Rightarrow f(0) = \frac{1}{2}$$

$$\Rightarrow f(x) + \frac{1}{2} = x^2 - 2x + 1 \Rightarrow f(x) = x^2 - 2x + \frac{1}{2}$$

$$\Rightarrow f(x^2) = x^4 - 2x^2 + \frac{1}{2}$$

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$$(x-1)y = x^2 + x + 2 \quad xy - y = x^2 + x + 2$$

$$x^2 + (1-y)x + y + 2 = 0 \Rightarrow x = \frac{-1(1-y) \pm \sqrt{(1-y)^2 - 4(y+2)}}{2}$$

$$\Rightarrow (1-y)^2 - 4(y+2) \geq 0 \Rightarrow y^2 + 1 - 2y - 4y - 8 \geq 0 \Rightarrow y^2 - 4y - 7 \geq 0$$

$$\Rightarrow (y-7)(y+1) \geq 0 \Rightarrow y \geq 7 \vee y \leq -1 \Rightarrow R = (-\infty, -1] \cup [7, +\infty)$$

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الف) $2y = 4x^2 - 10x + 2 \Rightarrow 2y = (2x - \frac{5}{4})^2 - \frac{25}{4} + 2 \Rightarrow x = \frac{\sqrt{2y + \frac{15}{2}} + \frac{5}{4}}{2} \Rightarrow y \geq -\frac{15}{4} \Rightarrow R = [-\frac{15}{4}, +\infty)$

ب) $y^2 = -(x^2 - 4x + 9 - 9 + 5) \Rightarrow y^2 = -(x-2)^2 + 4 \Rightarrow x = \sqrt{4 - y^2} + 2 \Rightarrow 4 - y^2 \geq 0 \Rightarrow -2 \leq y \leq 2 \Rightarrow R = [-2, 2]$

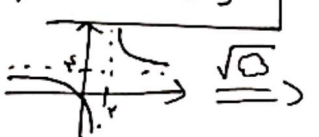
ج) $y = x^2 - 3x^2 + 4x + 1 \Rightarrow R = \mathbb{R} \Rightarrow y = \sqrt{x^2 - 3x^2 + 4x + 1} \Rightarrow R = [0, +\infty)$

د) $x^3 - 4x^2 + 7x + 4 \Rightarrow R = \mathbb{R}$

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الف)  $\Rightarrow R = \mathbb{R} - \{\frac{3}{2}\}$

ب) $y = \frac{x+1}{x-2}$  $\Rightarrow R = [0, +\infty) - \{1\}$

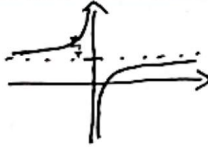
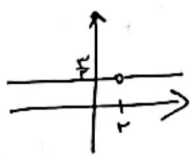
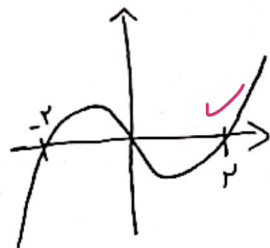
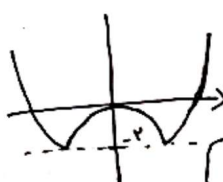
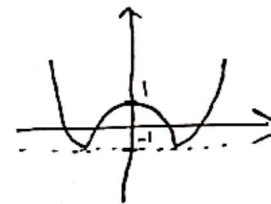
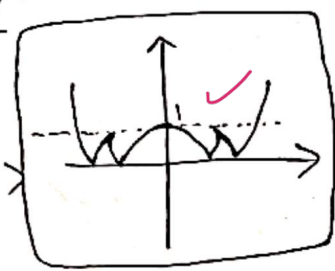
ج) $y = \sqrt{x^2 + 4} + \frac{1}{\sqrt{x^2 + 4}}$ $\Rightarrow y \geq 2 \Rightarrow R = [2, +\infty)$

د) $y = \frac{x^3 + 4}{x^2 + 4} - 4$ $\Rightarrow y + 4 \geq 2 \Rightarrow y \geq -2 \Rightarrow R = [-2, +\infty)$

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قابلیت یک شانه
!! نماند

ررسی اصلی: $y x^2 - y = 3x^2 + 4$ $x^2(y - 3) = y + 4$ $x^2 = \frac{y+4}{y-3} \rightarrow y \neq 3 \quad ①$ $\frac{y+4}{y-3} > 0 \Rightarrow y > 3 \vee y < -4 \quad ②$ تذکره: $0 < x^2 < +\infty$ $\Rightarrow x^2 = 0 \Rightarrow y = -4$ $x^2 = +\infty \Rightarrow y = 3$ حد ① و ② $\Rightarrow R = (-\infty, -4] \cup (3, +\infty)$ ✓ مخرج می تواند صفر شود $\Rightarrow R = (-\infty, -4] \cup (3, +\infty)$ ✓	۲ ۶
$-1 \leq \sin x \leq 1 \Rightarrow \begin{cases} \sin x = 1 \Rightarrow f(x) = -3 \\ \sin x = -1 \Rightarrow f(x) = -\frac{1}{4} \end{cases}$ مخرج نمی تواند صفر شود $\Rightarrow$ $R = [-3, -\frac{1}{4}]$ $R = [-3, -\frac{1}{4}]$ ✓	۲ ۷
الف) $y = \frac{x-1}{x^2}$ شکل  $\Rightarrow$  $\Rightarrow R = [0, +\infty)$ ✓ ب) $y = \sqrt{\frac{9(x-4)}{4(x-2)}} = \frac{3}{2}$  $\Rightarrow R = \{\frac{3}{2}\}$ ✓	۲ ۸
$y = \begin{cases} x^2 - 2x & x \geq 0 \\ -x^2 - 2x & x < 0 \end{cases} \Rightarrow$  $\Rightarrow R = \mathbb{R}$ ✓	۲ ۹
$y = f(x)$  $\Rightarrow y = f(x) + 1$  $y = f(x) + 1$ $\Rightarrow$ 	۲ ۱۰

$$\text{بـ) } S \Big|_r^r \Rightarrow R_y = \sqrt{(-\infty, r]} = [0, r]$$

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$$\text{جـ) } y = \frac{x^r + r + 1}{\sqrt{x^r + r}} = \sqrt{x^r + r} + \frac{1}{\sqrt{x^r + r}}$$

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$$\xrightarrow{a > 0} \min a = \sqrt{r} \rightarrow y_{\min} = \frac{r\sqrt{r}}{r} \Rightarrow R_y = \left[\frac{r\sqrt{r}}{r}, +\infty \right)$$

$$\text{دـ) } y = x^r + r + \frac{1}{x^r + r} - r \xrightarrow{a > 0} \min a = r \rightarrow y_{\min} = \frac{1}{r}$$

$$\Rightarrow R_y = \left[\frac{1}{r}, +\infty \right)$$