

$$x \Rightarrow -x \Rightarrow f(x) + x f(-x) = 2|x| + 1$$

$$f(-x) - x f(x) = 2|x| + 1 \xrightarrow{x(-x)} \begin{cases} x f(-x) + f(x) = 2|x| + 1 \\ -x f(-x) + x^2 f(x) = (-2|x| - 1)x \end{cases}$$

$$(x^2 + 1)f(x) = 2(1-x)|x| + 1 - x$$

$$\Rightarrow f(x) = \frac{2(1-x)|x| + 1 - x}{x^2 + 1}$$

1

$$x=0 \Rightarrow f(0) + f(0) = 0 - 2(0) + 1 \Rightarrow f(0) = \frac{1}{2}$$

$$\Rightarrow f(x) + \frac{1}{2} = x^2 - 2x + 1 \Rightarrow f(x) = x^2 - 2x + \frac{1}{2}$$

$$\Rightarrow f(x^2) = x^4 - 2x^2 + \frac{1}{2}$$

2

$$(x-1)y = x^2 + x + 2 \quad xy - y = x^2 + x + 2$$

$$x^2 + (1-y)x + y + 2 = 0 \Rightarrow x = \frac{-\sqrt{(1-y)^2 - 4(y+2)} + y - 1}{2}$$

$$\Rightarrow (1-y)^2 - 4(y+2) \geq 0 \Rightarrow y^2 + 1 - 2y - 4y - 8 \geq 0 \Rightarrow y^2 - 4y - 7 \geq 0$$

$$\Rightarrow (y-7)(y+1) \geq 0 \Rightarrow y \geq 7 \vee y \leq -1 \Rightarrow R = (-\infty, -1] \cup [7, +\infty)$$

3

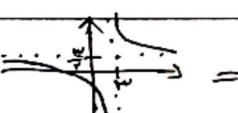
الف)  $2y = 4x^2 - 10x + 2 \Rightarrow 2y = (2x - \frac{5}{2})^2 - \frac{25}{4} + 2 \Rightarrow x = \frac{\sqrt{2y + \frac{13}{2}} + \frac{5}{2}}{2} \Rightarrow y \geq -\frac{13}{4} \Rightarrow R = [-\frac{13}{4}, +\infty)$

ب)  $y^2 = -(x^2 - 4x + 9 - 9 + 5) \Rightarrow y^2 = -(x-2)^2 + 4 \Rightarrow x = \sqrt{4 - y^2} + 2 \Rightarrow 4 - y^2 \geq 0 \Rightarrow -2 \leq y \leq 2 \Rightarrow R = [-2, 2]$

ج)  $y = x^2 - 3x^2 + 4x + 1 \Rightarrow R = \mathbb{R} \Rightarrow y = \sqrt{x^2 - 3x^2 + 4x + 1} \Rightarrow R = [0, +\infty)$

د)  $x^2 - 4x^2 + 7x + 4 \Rightarrow R = \mathbb{R}$   
 گاریم به 4 IR می دهیم

4

الف)   $\Rightarrow R = \mathbb{R} - \{\frac{3}{2}\}$

ب)  $y = \frac{x+1}{x-2}$    $\Rightarrow R = [0, +\infty) - \{1\}$

ج)  ~~$y = \sqrt{x^2 + 1} + \frac{1}{\sqrt{x^2 + 1}} \xrightarrow{\frac{1}{x^2 + 1} > 0} y \geq 2 \Rightarrow R = [2, +\infty)$~~

د)  $y = x^2 + 2 + \frac{1}{x^2 + 4} - 2 \xrightarrow{x^2 + 4 > 0} y + 2 \geq 2 \Rightarrow y \geq -2 \Rightarrow R = [-2, +\infty)$

5

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |    |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| <p>روش اصلی:</p> $y x^2 - y = 3x^2 + 4$ $x^2(y - 3) = y + 4$ $x^2 = \frac{y+4}{y-3} \rightarrow y \neq 3 \quad ①$ $\frac{y+4}{y-3} > 0 \Rightarrow y > 3 \vee y < -4 \quad ②$ <p>تذکره: <math>0 &lt; x^2 &lt; +\infty</math><br/> <math>\Rightarrow x^2 = 0 \Rightarrow y = -4</math><br/> <math>x^2 = +\infty \Rightarrow y = 3</math> حد</p> <p>① و ② <math>\Rightarrow R = (-\infty, -4] \cup (3, +\infty)</math><br/> مخرج می تواند صفر شود <math>\Rightarrow R = (-\infty, -4] \cup (3, +\infty)</math></p> | ۶  |
| $-1 \leq \sin x \leq 1 \Rightarrow \begin{cases} \sin x = 1 \Rightarrow f(x) = -3 \\ \sin x = -1 \Rightarrow f(x) = -\frac{1}{4} \end{cases}$ <p>مخرج نمی تواند صفر شود <math>\Rightarrow \sin x \neq 3</math></p> <p><del><math>R = \mathbb{R}</math></del> <math>R = [-3, -\frac{1}{4}]</math></p>                                                                                                                                                                                                             | ۷  |
| <p>الف) <math>y = \frac{x-1}{x^2}</math> شکل </p> <p><math>\Rightarrow R = [0, +\infty)</math></p> <p>ب) <math>y = \sqrt{\frac{9(x-4)}{4(x-2)}} = \frac{3}{2}</math> <math>\Rightarrow R = \{\frac{3}{2}\}</math></p>                                                                                                                                                                                                                                                                                            | ۸  |
| $y = \begin{cases} x^2 - 2x & x \geq 0 \\ -x^2 - 2x & x < 0 \end{cases}$ <p>شکل </p> <p><math>\Rightarrow R = \mathbb{R}</math></p>                                                                                                                                                                                                                                                                                                                                                                              | ۹  |
| <p><math>y = f( x )</math> </p> <p><math>y = f( x ) + 1</math> </p> <p><math>y =  f( x ) + 1 </math> </p>                                                                                                                                                                                                                                                                                                                                                                                                        | ۱۰ |