

$$f(x) - xf(x) = 2|x| + 1 \Rightarrow f(-x) = 2|x| + 1 + xf(x) \quad (1) \quad (2)$$

$$f(x) + xf(-x) = 2|x| + 1 \Rightarrow f(x) + x(2|x| + 1 + xf(x)) = 2|x| + 1 \Rightarrow$$

$$f(x) = \frac{(2|x| + 1)(1 - x^2)}{1 + x^2} \quad \checkmark \quad x \in \mathbb{R}$$

$$x = 0 \Rightarrow f(0) + f(0) = 2 \Rightarrow 2f(0) = 2 \Rightarrow f(0) = 1$$

$$f(x^2) + f(0) = (x^2)^2 - 2(x^2) + 2 \Rightarrow f(x^2) = x^4 - 2x^2 + 2 \quad \checkmark$$

$$y = \frac{x^2 + x + 2}{x - 1} \quad \text{عوض کنیم} \Rightarrow yx - y = x^2 + x + 2 \Rightarrow x^2 + (1 - y)x + y + 2 = 0 \quad (3) \quad (4)$$

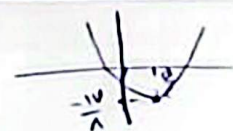
$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow y - 1 \pm \sqrt{(1 - y)^2 - 4(y + 2)} \Rightarrow y - 1 \pm \sqrt{y^2 - 2y - 7}$$

$$\sqrt{\Delta} \geq 0 \Rightarrow y^2 - 2y - 7 \geq 0 \Rightarrow (y + 1)(y - 7) \geq 0 \quad y \in (-\infty, -1] \cup [7, +\infty)$$

$$R_f = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف) } y = 2x^2 - 5x + 1 \quad \text{بمیانگین} \quad -\frac{b}{2a} = \left[\frac{5}{4} = x \right] \Rightarrow y = -\frac{17}{8}$$

$$R_f = \left[-\frac{17}{8}, +\infty \right) \quad \checkmark$$



$$\text{ب) } y = \sqrt{-x^2 + 4x - 5} \rightarrow -x^2 + 4x - 5 \leq 0 \Rightarrow y = 4$$

$$y = \sqrt{(-\infty, 4]} \Rightarrow R = [0, 4] \quad \checkmark$$



$$\text{ج) } \sqrt{x^2 - 2x^2 + 4x + 1} \Rightarrow$$

$$\sqrt{(-\infty, +\infty)} \Rightarrow [0, +\infty) = R_y$$

$$\text{د) } y = \lg(x^2 - 4x^2 + 7x + 4) \Rightarrow x^2 - 4x^2 + 7x + 4 \rightarrow R = R_y$$

$$\lg(-\infty, +\infty) \Rightarrow R_y = (-\infty, +\infty) \quad \checkmark \quad \mathbb{R}$$

(۵) در تمام موارد به شکل $R = \{a\}$ است.

الف) $\frac{x+1}{x-2} \Rightarrow R_y = \mathbb{R} - \left\{\frac{1}{2}\right\}$

(۲)

ب) $\sqrt{\frac{x+1}{x-2}} \Rightarrow R_y = \mathbb{R} - \{0\} \Rightarrow R_y = [0, +\infty) - \{0\}$

ج) $\frac{x^2+4}{\sqrt{x^2+4}} \Rightarrow \frac{x^2+4+1}{\sqrt{x^2+4}} \Rightarrow \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \xrightarrow{a+\frac{1}{a}} \sqrt{4} + \frac{1}{\sqrt{4}} = \frac{4\sqrt{4}}{4}$
 $\sqrt{x^2+4}$ نمی تواند ۰ شود، کمترین مقدار آن $\sqrt{4}$ است $\Leftarrow$
 $R_y = \left[\frac{4\sqrt{4}}{4}, +\infty\right)$

د) $y = x^2 + \frac{1}{x^2+4} \Rightarrow x^2+4 + \frac{1}{x^2+4} - 4 \xrightarrow{a+\frac{1}{a}} 4 + \frac{1}{4} - 4 = \left(\frac{1}{4}\right)$
 x^2+4 نمی تواند ۰ شود، کمترین مقدار آن $\frac{1}{4}$ است $\Leftarrow$
 $R_y = \left[\frac{1}{4}, +\infty\right)$

۶) روشی اصلی $\Leftarrow$
 $y = \frac{x^2+4}{x^2-1} \Rightarrow yx^2 - y = x^2+4 \Rightarrow yx^2 - x^2 = y+4 \Rightarrow x^2(y-1) = y+4 \Rightarrow x^2 = \frac{y+4}{y-1} \Rightarrow x = \pm \sqrt{\frac{y+4}{y-1}} \Rightarrow \frac{y+4}{y-1} \geq 0$
 $\frac{-4}{-1} < \frac{y}{y-1} < \frac{4}{1} \Rightarrow R_y = (-\infty, -1) \cup (5, +\infty)$

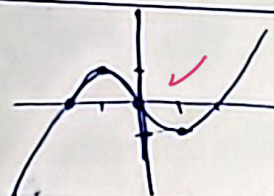
۷. $x^2 = \frac{y+4}{y-1} \Rightarrow \frac{y+4}{y-1} \geq 0 \rightarrow y \leq -1$
 $\rightarrow y \geq 5$

۷) $f(x) = \frac{x \sin x + 4}{\sin x - 1} \Rightarrow y \sin x - y = x \sin x + 4 \Rightarrow \sin x (y-1) = y+4$
 $\sin x = \frac{y+4}{y-1} \Rightarrow -1 \leq \sin x \leq 1 \Rightarrow -1 \leq \frac{y+4}{y-1} \leq 1$
 $\left. \begin{array}{l} \frac{y+4}{y-1} \leq 1 \\ \frac{y+4}{y-1} \geq -1 \end{array} \right\} \begin{array}{l} y \leq -\frac{1}{2} \\ y \geq -3 \end{array} \Rightarrow R_y = \left[-3, -\frac{1}{2}\right]$

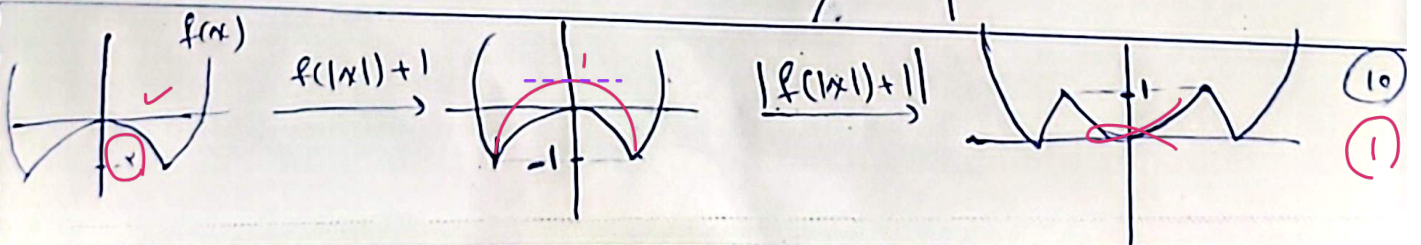
الف) $y = \left| \frac{x-1}{x} \right| \rightarrow x \neq 0 \Rightarrow \left| \frac{x}{x} - \frac{1}{x} \right|$
 $R_y = [0, +\infty)$

ب) $\sqrt{\frac{2x-11}{-1+4x}} \Rightarrow \sqrt{\frac{2(x-2)}{4(x-2)}} \Rightarrow \sqrt{\frac{1}{2}} = \left\{\frac{\sqrt{2}}{2}\right\} \Rightarrow R_y = \left\{\frac{\sqrt{2}}{2}\right\}$

$y = x(|x|-2) \Rightarrow \begin{cases} x \geq 0 & x^2 - 2x \\ x < 0 & -x^2 - 2x \end{cases} \Rightarrow R_y = \mathbb{R}$



(۲) (۹)



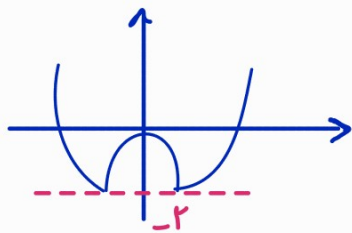
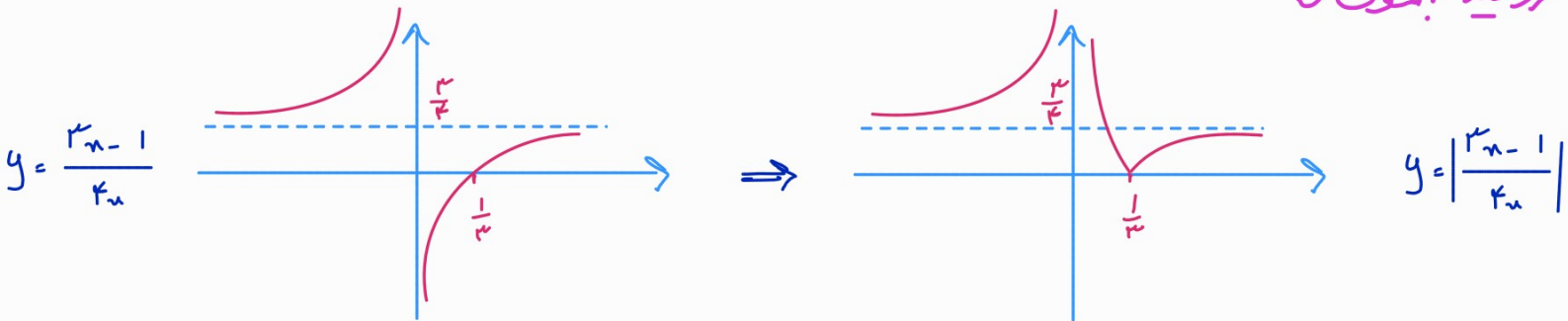
(۱۰) (۱)

۸- اول برد عبارت داخل قدر مطلق
رو حساب می کنیم بعد قدر مطلق می گیریم

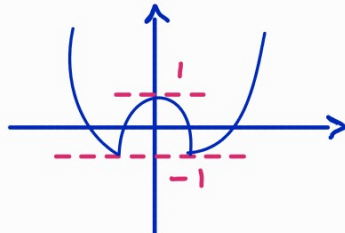
فقط خواست باشد که $y = \frac{3}{4}$ هم هست داخل برد

$y = -\frac{3}{4}$ عضو برد عبارت داخل قدر مطلق هست و وقتی قدر مطلق بگیریم $y = \frac{3}{4}$

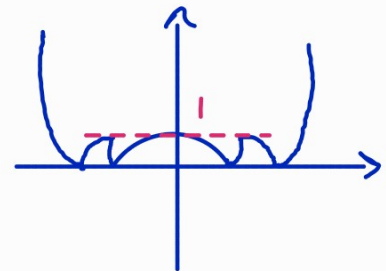
روید به نمودار:



$$y = f(1-x)$$



$$y = f(|x|) + 1$$



$$y = |f(|x|) + 1|$$

۱۰-