

۲. خیلی خوب (نویسنده: افشین)

$$f(-x) - xf(x) = 2|x| + 1 \xrightarrow{x=-x} f(x) + xf(-x) = 2|x| + 1$$

$$\begin{cases} f(-x) - xf(x) = 2|x| + 1 \\ f(x) + xf(-x) = 2|x| + 1 \end{cases} \Rightarrow \begin{cases} xf(-x) - xf(x) = 2|x| + x \\ -xf(-x) + f(x) = 2|x| + 1 \end{cases}$$

$$-x^2 f(x) - f(x) = 2|m| + x - 2|m| - 1$$

$$\hookrightarrow f(x)(-x^2 - 1) = 2|m| + x - 2|m| - 1 \Rightarrow f(x) = \frac{2|m| + x - 2|m| - 1}{-x^2 - 1}$$

$$f(x) = \begin{cases} \frac{2x^2 - x - 1}{-x^2 - 1}, & x \geq 0 \\ \frac{-2x^2 + 2x - 1}{-x^2 - 1}, & x < 0 \end{cases}$$

$$f(x) + f(0) = x^2 - 2x + 4 \xrightarrow{x=0} 2f(0) = 4 \Rightarrow f(0) = 2$$

$$\hookrightarrow f(x) + 2 = x^2 - 2x + 4 \Rightarrow f(x) = x^2 - 2x + 2$$

$$\rightarrow f(x^2) = (x^2)^2 - 2(x^2) + 2 = x^4 - 2x^2 + 2$$

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow yx - y = x^2 + x + 2 \rightarrow -x^2 + yx - x - y - 2 = 0 \rightarrow -x^2 + (y-1)x + (-y-2) = 0$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow x = \frac{-(y-1) \pm \sqrt{(y-1)^2 - 4(-1)(-y-2)}}{-2} \Rightarrow (y-1)^2 - 4(-1)(-y-2) \geq 0 \rightarrow y^2 - 2y + 1 - 4y - 8 \geq 0$$

$$\rightarrow y^2 - 4y - 7 \geq 0 \Rightarrow (y+1)(y-7) \geq 0$$

$$\begin{matrix} y = -1 \\ y = 7 \end{matrix} \quad \begin{matrix} - & + & - & + & - \\ + & - & + & - & + \end{matrix} \Rightarrow R_y = (-\infty, -1] \cup [7, +\infty)$$

$$\text{الف) } y = 2x^2 - 5x + 1 \rightarrow y_{\min} = \frac{-\Delta}{4a} = \frac{-(25 - 4(2)(1))}{4} = \frac{-17}{4} \Rightarrow R_y = \left[-\frac{17}{4}, +\infty\right)$$

$$\text{ب) } y = \sqrt{-x^2 + 4x - 5} \rightarrow y_{\max} = \frac{-\Delta}{4a} = \frac{-(4 - 4(-1)(-5))}{-4} = 4 \Rightarrow R_y = \sqrt{(-\infty, 4]} \rightarrow R_y = [0, 4]$$

$$\text{ج) } y = \sqrt{x^2 - 4x^2 + 4x + 1} \rightarrow R_y = \sqrt{1R} \Rightarrow R_y = [0, +\infty)$$

$$\text{د) } y = \log(x^2 - 4x^2 + 4x + 5) \rightarrow R_y = (-\infty, +\infty)$$

$$\text{الف) } y = \frac{x+1}{x-4} \xrightarrow{\text{تجزیه کسر}} R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} \Rightarrow R_y = \mathbb{R} - \left\{\frac{1}{4}\right\}$$

$$\text{ب) } y = \sqrt{\frac{x+1}{x-2}} \Rightarrow R_y = \sqrt{1R - \left\{\frac{a}{c}\right\}} \Rightarrow R_y = [0, +\infty) - \left\{\frac{1}{4}\right\}$$

$$\text{ج) } y = \frac{x^2 + 4}{\sqrt{x^2 + 3}} \rightarrow y = \frac{x^2 + 4}{\sqrt{x^2 + 3}} \Rightarrow y = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \xrightarrow{\min} \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{3} \Rightarrow R_y = \left[\frac{4\sqrt{3}}{3}, +\infty\right)$$

$$\text{د) } y = x^2 + \frac{1}{x^2 + 4} \rightarrow y = x^2 + \frac{1}{x^2 + 4} + 4 - 4 \rightarrow y = x^2 + 4 + \frac{1}{x^2 + 4} - 4 \xrightarrow{\min} 4 + \frac{1}{4} - 4 = \frac{1}{4} \Rightarrow R_y = \left[\frac{1}{4}, +\infty\right)$$

$$\text{مثال ۲: } y = \frac{x^2 + 4}{x^2 - 1}$$

$$y x^2 - y = x^2 + 4 \Rightarrow y x^2 - x^2 = y + 4 \Rightarrow x^2(y - 1) = y + 4 \Rightarrow x^2 = \frac{y + 4}{y - 1} \rightarrow x = \sqrt{\frac{y + 4}{y - 1}} \rightarrow \frac{y + 4}{y - 1} \geq 0$$

$$\frac{-4}{1} < \frac{y}{-1} < \frac{4}{1} \rightarrow R_y = (-\infty, -4] \cup (4, +\infty)$$

$$\text{نکته: } x^2 \begin{cases} \min = 0 \\ \max = 100 \end{cases} \rightarrow \begin{cases} \frac{0 + 4}{0 - 1} = -4 \\ \frac{100 + 4}{100 - 1} \approx 4 \end{cases} \rightarrow R_y = (-\infty, -4] \cup (4, +\infty)$$

مخرج می تواند صفر شود پس بر مخرج این دقت داشته باشید

۶

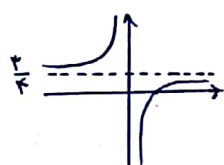
$$f(x) = \frac{x \sin x + 4}{\sin x - 3}$$

$$\sin x \begin{cases} \min = -1 \\ \max = 1 \end{cases} \rightarrow \begin{cases} \frac{-1 + 4}{-1 - 3} = -\frac{1}{2} \\ \frac{1 + 4}{1 - 3} = -\frac{5}{2} \end{cases}$$

$$\text{چون مخرج نمی تواند صفر شود، بر مخرج این دقت داشته باشید} \rightarrow R_y = [-\frac{5}{2}, -\frac{1}{2}]$$

۷

$$\text{مثال ۳: } y = \left| \frac{x^2 - 1}{x^2} \right|$$



$$y = \frac{x^2 - 1}{x^2} \begin{cases} \text{محاسبه مشتق} = 0 \\ \text{محاسبه انتی} = \frac{x}{x} \end{cases}$$

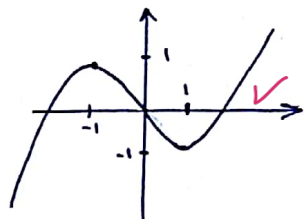
$$y = \left| \frac{x^2 - 1}{x^2} \right| \rightarrow R_y = [0, +\infty)$$

۸

$$\text{ب) } y = \sqrt{\frac{9x - 11}{-1 + 6x}} \rightarrow y = \sqrt{\frac{9x - 11}{6x - 1}} \rightarrow y = \sqrt{\frac{9(x - \frac{11}{9})}{6(x - \frac{1}{6})}} \Rightarrow y = \sqrt{\frac{3}{2}} \rightarrow y = \frac{\sqrt{6}}{2}$$

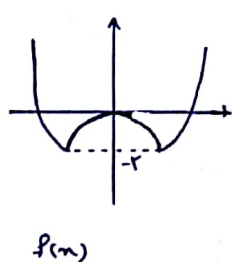
$$\Rightarrow R_y = \left\{ \frac{\sqrt{6}}{2} \right\}$$

$$y = x|x| - 2x \begin{cases} x \geq 0 \rightarrow y = x^2 - 2x \rightarrow \text{ext} \left| \begin{array}{l} -\frac{b}{2a} = 1 \\ 1 - 2 = -1 \end{array} \right. \\ x < 0 \rightarrow y = -x^2 - 2x \rightarrow \text{ext} \left| \begin{array}{l} -\frac{b}{2a} = -1 \\ -1 + 2 = 1 \end{array} \right. \end{cases}$$

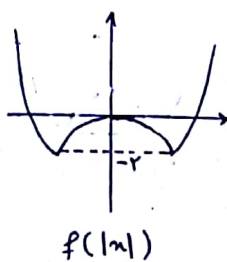


$$R_y = \mathbb{R}$$

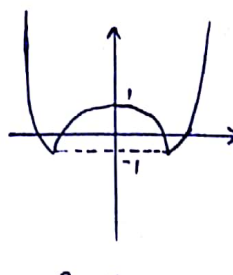
۹



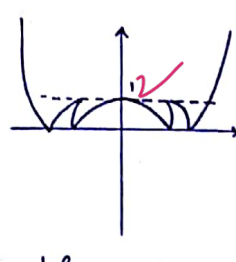
$$f(x)$$



$$f(|x|)$$



$$f(|x|) + 1$$



$$|f(|x|) + 1|$$

۱۰