

$$f(-x) - xf(x) = r|x| + 1 \xrightarrow{x=-x} f(x) + xf(-x) = r|x| + 1$$

$$\begin{cases} f(-x) - xf(x) = r|x| + 1 \\ f(x) + xf(-x) = r|x| + 1 \end{cases} \Rightarrow \begin{cases} xf(-x) - xf(x) = r|x| + x \\ -xf(-x) + f(x) = r|x| + 1 \end{cases}$$

$$-x^r f(x) - f(x) = r|m| + x - r|m| - 1$$

$$\hookrightarrow f(x)(-x^r - 1) = r|m| + x - r|m| - 1 \Rightarrow f(x) = \frac{r|m| + x - r|m| - 1}{-x^r - 1}$$

$$f(x) = \begin{cases} \frac{rx^r - x - 1}{-x^r - 1}, & x \geq 0 \\ \frac{-rx^r + x - 1}{-x^r - 1}, & x < 0 \end{cases}$$

$$f(x) + f(0) = x^r - rx + 4 \xrightarrow{x=0} rf(0) = 4 \Rightarrow f(0) = 4$$

$$\hookrightarrow f(x) + 4 = x^r - rx + 4 \Rightarrow f(x) = x^r - rx + 4$$

$$\rightarrow f(x^r) = (x^r)^r - r(x^r) + 4 = \boxed{f(x^r) = x^{r^2} - rx^r + 4}$$

$$y = \frac{x^r + x + r}{x - 1} \rightarrow yx - y = x^r + x + r \rightarrow -x^r + yx - x - y - r = 0 \rightarrow -x^r + (y-1)x + (-y-r) = 0$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow x = \frac{-(y-1) \pm \sqrt{(y-1)^2 - 4(-1)(-y-r)}}{-2} \Rightarrow (y-1)^2 - 4(-1)(-y-r) \geq 0 \Rightarrow y^2 - 2y + 1 - 4y - 4r \geq 0$$

$$\rightarrow y^2 - 4y - 3 \geq 0 \Rightarrow (y+1)(y-5) \geq 0$$

$$\begin{matrix} y = -1 \\ y = 5 \end{matrix} \Rightarrow \boxed{R_y = (-\infty, -1] \cup [5, +\infty)}$$

$$\text{ا) } y = rx^r - x + 1 \rightarrow y_{\min} = \frac{-\Delta}{2a} = \frac{-(r^2 - 4r)(1)}{2r} = \frac{-r}{2} \Rightarrow \boxed{R_y = [-\frac{r}{2}, +\infty)}$$

$$\text{ب) } y = \sqrt{-x^r + 4x - 5} \rightarrow y_{\max} = \frac{-\Delta}{2a} = \frac{-(4 - 4(-1)(-5))}{-2} = 4 \Rightarrow \boxed{R_y = (-\infty, 4] \rightarrow R_y = [0, 2]}$$

$$\text{ج) } y = \sqrt{x^r - rx^r + rx + 1} \rightarrow R_y = \sqrt{\mathbb{R}} \Rightarrow \boxed{R_y = [0, +\infty)}$$

$$\text{د) } y = \log(x^r - 4x^r + rx + 5) \rightarrow \boxed{R_y = (-\infty, +\infty)}$$

$$\text{ا) } y = \frac{rx+1}{rx-4} \xrightarrow{\text{تجزیه}} R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} \Rightarrow \boxed{R_y = \mathbb{R} - \left\{\frac{r}{4}\right\}}$$

$$\text{ب) } y = \sqrt{\frac{rx+1}{x-2}} \Rightarrow R_y = \sqrt{\mathbb{R} - \{2\}} \Rightarrow \boxed{R_y = [0, +\infty) - \{2\}}$$

$$\text{ج) } y = \frac{x^r + r}{\sqrt{rx^r + r}} \rightarrow y = \frac{x^r + r + 1}{\sqrt{rx^r + r}} \xrightarrow{\text{min}} \sqrt{r} + \frac{1}{\sqrt{r}} - \frac{r\sqrt{r}}{r} \Rightarrow \boxed{R_y = \left[\frac{r\sqrt{r}}{r}, +\infty\right)}$$

$$\text{د) } y = x^r + \frac{1}{x^r + r} \rightarrow y = x^r + \frac{1}{x^r + r} + t - t \rightarrow y = x^r + t + \frac{1}{x^r + r} - t \xrightarrow{\text{min}} t + \frac{1}{t} - t = \frac{1}{t} \Rightarrow \boxed{R_y = \left[\frac{1}{r}, +\infty\right)}$$

$$\text{مثال ۶: } y = \frac{x^2 + 4}{x^2 - 1}$$

$$y x^2 - y = x^2 + 4 \Rightarrow y x^2 - x^2 = y + 4 \Rightarrow x^2(y - 1) = y + 4 \Rightarrow x^2 = \frac{y + 4}{y - 1} \rightarrow x = \sqrt{\frac{y + 4}{y - 1}} \rightarrow \frac{y + 4}{y - 1} \geq 0$$

$$\frac{-4}{1} < \frac{y}{-1} < \frac{4}{1} \rightarrow R_y = (-\infty, -1] \cup (1, +\infty)$$

$$\text{نکته: } x^2 \begin{cases} \min = 0 \\ \max = 100 \end{cases} \rightarrow \begin{cases} \frac{0+4}{0-1} = -4 \\ \frac{100+4}{100-1} \approx 1 \end{cases} \rightarrow R_y = (-\infty, -1] \cup (1, +\infty)$$

مخرج می تواند صفر شود پس باید مخرج این رادیکال را

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$$f(x) = \frac{x \sin x + 1}{\sin x - 1}$$

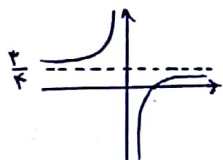
$$\sin x \begin{cases} \min = -1 \\ \max = 1 \end{cases} \rightarrow \begin{cases} \frac{-1+1}{-1-1} = -\frac{1}{2} \\ \frac{1+1}{1-1} = -\infty \end{cases}$$

$$\text{چون مخرج نمی تواند صفر شود،} \rightarrow R_y = [-\frac{1}{2}, -\infty)$$

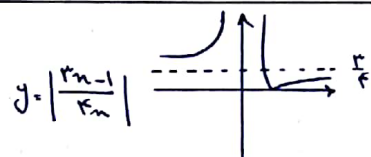
بر مبنای این رادیکال

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$$\text{مثال ۸: } y = \left| \frac{x-1}{x} \right|$$



$$y = \frac{x-1}{x} \begin{cases} \text{محدوده } y = 0 \\ \text{محدوده } y = \frac{1}{x} \end{cases}$$



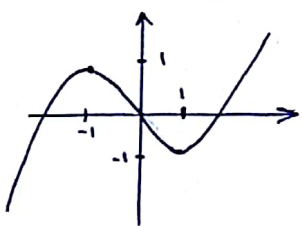
$$R_y = [0, +\infty)$$

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$$\text{ب) } y = \sqrt{\frac{9x-11}{-1+x}} \rightarrow y = \sqrt{\frac{9x-11}{x-1}} \rightarrow y = \sqrt{\frac{9(x-1)}{x-1}} \Rightarrow y = \sqrt{\frac{9}{1}} \rightarrow y = \frac{3}{1}$$

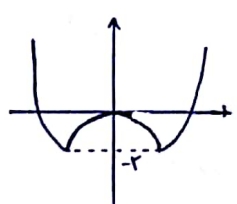
$$\Rightarrow R_y = \left\{ \frac{3}{1} \right\}$$

$$y = x|x| - 2x \begin{cases} x \geq 0 \rightarrow y = x^2 - 2x \rightarrow \text{ext} \left| \begin{matrix} -\frac{b}{2a} = 1 \\ 1-2 = -1 \end{matrix} \right. \\ x \leq 0 \rightarrow y = -x^2 - 2x \rightarrow \text{ext} \left| \begin{matrix} -\frac{b}{2a} = -1 \\ -1+2 = 1 \end{matrix} \right. \end{cases}$$

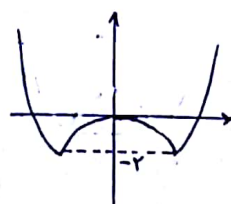


$$R_y = \mathbb{R}$$

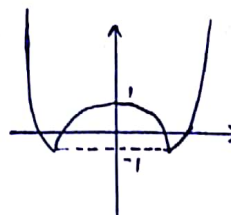
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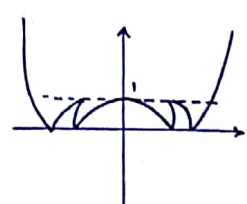
$$f(x)$$



$$f(|x|)$$



$$f(|x|) + 1$$



$$|f(|x|) + 1|$$

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