

الف) $y = \frac{|n|+1}{|n|-r}$ $y/|n| - ry = |n|+1 \Rightarrow y/|n| - |n| = 1+ry$ $|n|(y-1) = 1+ry$ (1)
 $|n| = \frac{1+ry}{y-1} \rightarrow y = -\frac{1}{r}$ $\int \frac{1}{x} dx = \ln|x| + C$ $\frac{-1}{r} = \frac{1}{r} \Rightarrow R = (-\infty, -\frac{1}{r}] \cup (1, +\infty)$
 $\rightarrow y = \frac{r^2+1}{|n|}$ $y = \frac{|n|+1}{|n|} \Rightarrow \ln|n| + \frac{1}{|n|} \rightarrow \ln|n| > 0 \Rightarrow R = [r, +\infty)$

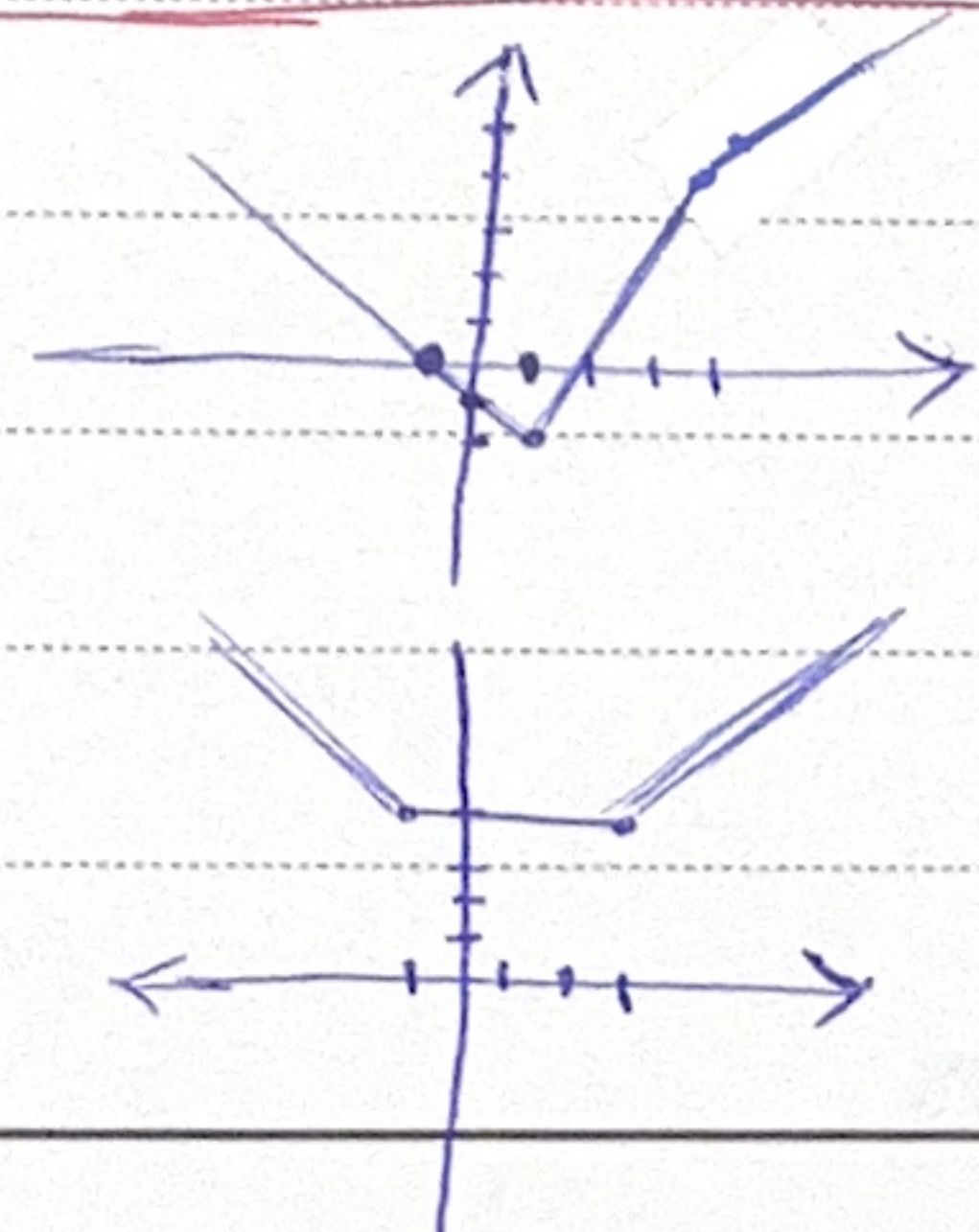
$y = \frac{n^2+rn-r}{n+r}$ $yn+ry = n^2+rn-r \Rightarrow n^2-yn+rn-ry-r=0 \Rightarrow n^2+(r-y)n-ry-r=0$ (2)
 $\Rightarrow n = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow \Delta > 0$ $b^2-4ac > 0 \rightarrow (r-y)^2 - (4(-ry-r)) = r^2+y^2-ry+1$
 $= y^2+1y+1 > 0$ $(y+r)(y+1) > 0$ $\frac{-4}{-4} = \frac{-r}{-4} \Rightarrow Ry = (-\infty, -4] \cup [-r, +\infty)$

$f(n) = \frac{n^2-n+\frac{1}{r}}{n^2-n+1}$ $Df \Rightarrow n^2-n+1 \neq 0 \rightarrow \Delta = 1-(4 \times 1 \times \frac{1}{r}) = -\frac{4}{r} < 0 \rightarrow \lim_{n \rightarrow \pm\infty} f(n) = \frac{1}{r}$ (3)
 $Rf \Rightarrow yn^2-yn+y = n^2-n+\frac{1}{r} \Rightarrow yn^2-n^2-yn+n+y-\frac{1}{r}=0 \Rightarrow n^2(y-1)+(1-y)n+y-\frac{1}{r}=0$
 $\Delta > 0$ $y^2+1-ry-(r(y-1)(y-\frac{1}{r})) = -ry(y-1) > 0$
 ① $rn = ry-r$ ② $\frac{0}{-4} = \frac{1}{-4} \Rightarrow Ry = [0, 1)$

الف) $y = \sqrt{-n^2+kn-1}$ $y' = -n^2+kn-1$ $\text{ext} \left| \frac{-b}{2a} = n \Rightarrow \frac{-k}{-2} = r \right.$ (4)
 $\rightarrow a < 0 \rightarrow \max$ $Ry' = (-\infty, r] \rightarrow Ry = [0, \sqrt{r}]$
 $\rightarrow y = \sqrt{kn^2-n}$ $y' = kn^2-n \rightarrow \text{ext} \left| n = \frac{-b}{2a} = \frac{1}{2k} = r \right.$ $a > 0 \rightarrow \min$
 $Ry' = [-1, +\infty) \rightarrow Ry = [-r, +\infty)$

الف) $y = \log(n^2-n+1)$ $y' = n^2-n+1$ $R = \mathbb{R}$ $(0, +\infty) \rightarrow \lim_{n \rightarrow \pm\infty} \log(x) = \pm\infty$ (5)
 $\Rightarrow \log: \mathbb{R} \rightarrow (-\infty, +\infty) \rightarrow Ry = (-\infty, +\infty) = \mathbb{R}$
 $\rightarrow \log_r(\frac{n^2+1}{n})$ $y' = \frac{n^2+1}{n} \Rightarrow y' > \frac{1}{n}$ $\lim_{n \rightarrow \pm\infty} \log(x) = \pm\infty$ $[r, +\infty) \rightarrow Ry = [1, +\infty)$

الف) $y = |2n-r| - |n-r|$ $\frac{-kn+r+n-r}{-n-1}$ $\frac{rn-r+n-r}{rn-\infty}$ $\frac{kn-r-n+r}{n+1}$ (6)
 $Ry = [-r, +\infty)$
 $\rightarrow y = |n-r| + |n+1|$ $\lim_{n \rightarrow \pm\infty} y = \infty$
 $|r-(-1)| = k$
 $Ry = [k, +\infty)$



$$|n-r| - |kn+r| \leq 1$$

$$\begin{array}{c|c|c} -n+r+kn+r & -n+r-kn-r & n-r-kn-r \\ \hline n+r \leq 1 & -kn \leq 1 & -n-r \leq 1 \end{array}$$

$$n \leq -1 \rightarrow n+r \leq 1 \Rightarrow n \leq -r \quad n \leq -1 \wedge n \leq -r \rightarrow n \leq -r \quad (1)$$

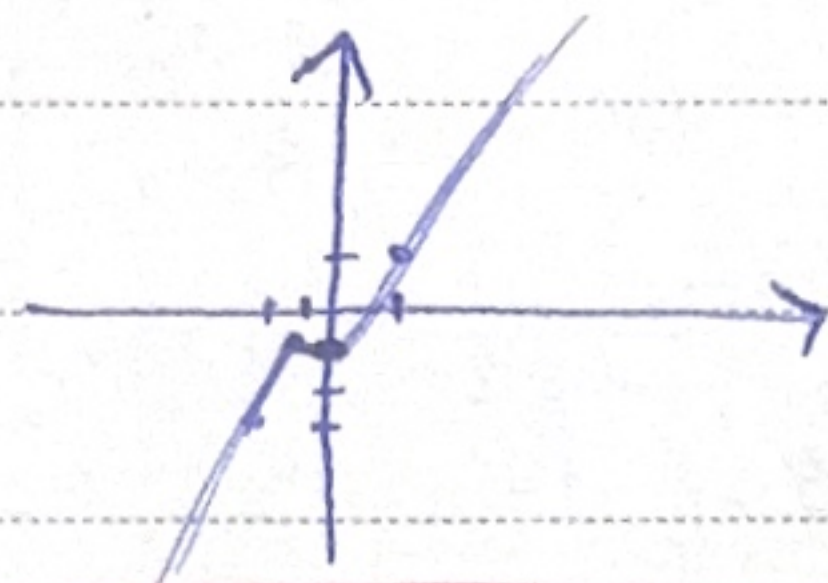
$$-1 \leq n \leq r \rightarrow -kn \leq 1 \quad n \geq -\frac{1}{k} \quad -1 \leq n \leq r \wedge n \geq -\frac{1}{k} \Rightarrow -\frac{1}{k} \leq n \leq r \quad (2)$$

$$n \geq r \rightarrow -n-r \leq 1 \quad -n \leq 1 \quad n \geq -1 \rightarrow n \geq r \quad (3)$$

$$(1) \vee (2) \vee (3) \rightarrow (-\infty, -r] \vee [-\frac{1}{k}, +\infty)$$

$$y = kn - |kn+1| + |n|$$

$$\begin{array}{c} n = -\frac{1}{k} \quad n = 0 \\ \hline kn+1 \quad -1 \quad 0 \quad kn-1 \end{array}$$



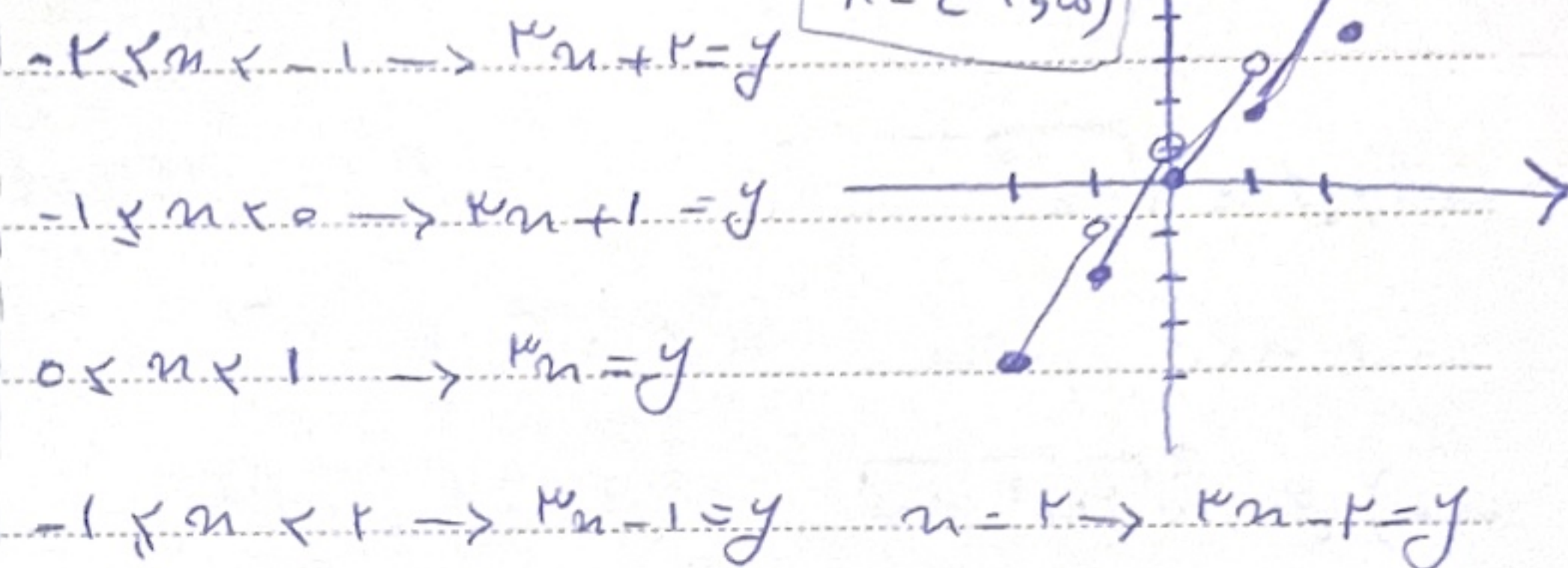
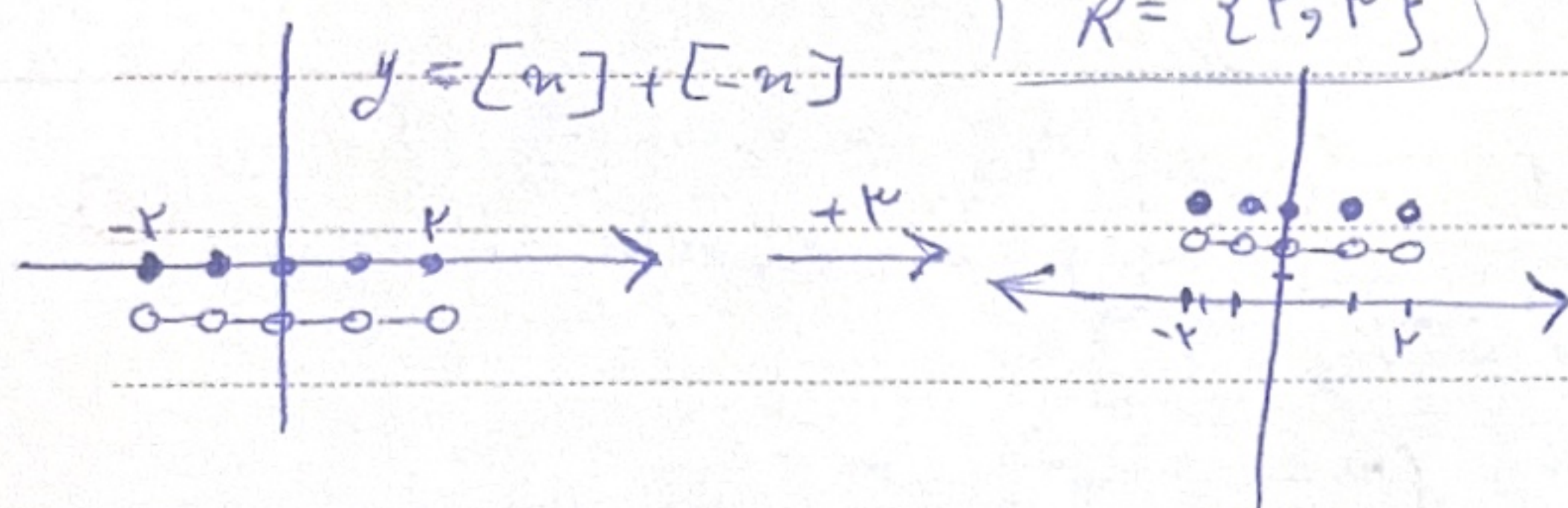
$$R_f = \mathbb{R}$$

$$\text{اذا } f = [n] + [r-n] = [n] + [-n] + k$$

$$\rightarrow y = kn - [n]$$

$$R = \{r, k\}$$

$$R = [-r, a]$$



$$\text{اذا } y = \sin^k n - \sin n + 1 \quad -1 \leq \sin n \leq 1$$

$$\xrightarrow{-1} (-1)^k - (-1) + 1 = k$$

$$R = [\frac{k}{k}, k]$$

$$\xrightarrow{\frac{b}{ka} = \frac{1}{k}} (\frac{1}{k})^k - \frac{1}{k} + 1 = \frac{k}{k}$$

$$\rightarrow y = \sin^k n + \sin^k n + 1 \quad 0 \leq \sin^k n \leq 1$$

$$\xrightarrow{0} 0 + 0 + 1 = 1$$

$$\xrightarrow{1} 1 + 1 + 1 = k$$

$$R = [1, k]$$

$$\xrightarrow{\frac{b}{ka} = \frac{1}{k}} \frac{1}{k} + \frac{1}{k} + 1 = \frac{k}{k}$$