

$$y = \frac{|x|+1}{|x|-2} \rightarrow y|x|-2y = |x|+1 \rightarrow |x|(y-1) = 2y+1 \rightarrow |x| = \frac{2y+1}{y-1} \quad (1) \quad (2)$$

$$\frac{2y+1}{y-1} \geq 0 \rightarrow (-\infty, -\frac{1}{2}] \cup (1, +\infty) = R_y$$

$$b) y = \frac{x^2+1}{|x|} \rightarrow \frac{x^2}{|x|} + \frac{1}{|x|} = \frac{|x|}{1} + \frac{1}{|x|} \rightarrow a > 0 \rightarrow a + \frac{1}{a} \geq 2 \rightarrow R_y = [2, +\infty)$$

$$y = \frac{x^2+2x-2}{x+1} \rightarrow xy + 2y = x^2+2x-2 \rightarrow x^2 + (1-y)x - 2y - 2 = 0 \quad (2) \quad (2)$$

$$(1-y)^2 - 4(-2y-2) \geq 0 \rightarrow 1 - 2y + y^2 + 8y + 8 = y^2 + 6y + 9 = (y+3)^2 \geq 0$$

$$\frac{-9}{+1} \leq y \leq \frac{-3}{+1} \rightarrow R_f: (-\infty, -3] \cup [-3, +\infty)$$

$$t \frac{x^2-x+1}{x^2-x+1} = \frac{t+1}{t+1} \quad x^2-x = (x-\frac{1}{2})^2 - \frac{1}{4} \quad \Delta K \text{ در } R \text{ در } (2) \quad (3)$$

$$t \in [-\frac{1}{4}, +\infty) \rightarrow \frac{t+1}{t+1} = \frac{(t+1) - \frac{1}{4}}{t+1} = 1 - \frac{\frac{1}{4}}{t+1} \quad t = -\frac{1}{4} \rightarrow 1$$

$$R \in [0, +\infty)$$

$$a) \sqrt{-x^2+4x-1} \rightarrow -x^2+4x-1 \geq 0 \rightarrow x^2-4x+1 \leq 0 \rightarrow x \in [2-\sqrt{3}, 2+\sqrt{3}] \quad (4) \quad (2)$$

$$\frac{2-\sqrt{3}}{+1} \leq y \leq \frac{2+\sqrt{3}}{+1} \rightarrow y_{\min} = 0, y_{\max} = y_s$$

$$x_s = 2 \rightarrow y_s = 3 \rightarrow -x^2+4x-1 \leq 3 \rightarrow -x^2+4x-4 \leq 0 \rightarrow x^2-4x+4 \geq 0 \rightarrow (x-2)^2 \geq 0$$

$$\rightarrow R [0, 3] \quad b) y = \sqrt[3]{x^2-4x} \rightarrow x_s = 2 \rightarrow y_s = -2 \rightarrow x^2-4x \geq -8 \rightarrow x^2-4x+8 \geq 0$$

$$\sqrt[3]{x^2-4x} \geq -2 \rightarrow R_y = [-2, +\infty)$$

همیشه مثبت؟

$$a) y = \log(x^2-x+1) \rightarrow x^2-x+1 > 0 \rightarrow R_y = \mathbb{R} \quad (5) \quad (1, 5)$$

$$b) \log\left(\frac{x^2+1}{x}\right) \rightarrow \frac{x^2+1}{x} > 0 \rightarrow D_y (0, +\infty)$$

$$x + \frac{1}{x} \xrightarrow{x>0} x + \frac{1}{x} \geq 2 \rightarrow \min = 2 \rightarrow R_y [\log 2, +\infty)$$

$$R_1 = [1, +\infty)$$

که زیاده ششون بدنه

برد توابع چند جمله ای از مرتبه فرد برابر IR می باشد:

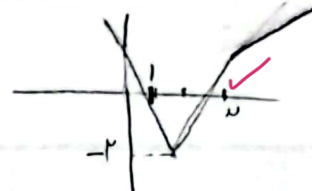
$$R_y = \log(-\infty, +\infty) = \log(0, +\infty) = (-\infty, +\infty) = \mathbb{R}$$



الف) $y = |x+1| - |x-3|$

$R = [-2, +\infty)$

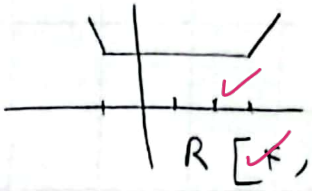
$\begin{array}{c} +1 \quad +3 \\ -x-1 \quad |x-3| \quad x+1 \end{array}$



(2) (6)

ب) $y = |x-1| + |x+1|$

$\begin{array}{c} -1 \quad 1 \\ -x+1 \quad |x+1| \quad x+1 \end{array}$



$R = [2, +\infty)$

$|x+1| - |x-1| \leq 1$

$\begin{array}{c} -1 \quad 1 \\ x+1 \quad |x-1| \quad x-1 \end{array}$

$x \leq -3 \rightarrow x < -1 \rightarrow x \leq 3$

$x > -1 \rightarrow x > 1 \rightarrow x > 3$

$x > 3 \rightarrow x > 1 \rightarrow x > 3$

$I \cap II \cap III = (-\infty, -3] \cup [-\frac{1}{2}, +\infty)$

(2) (7)

$y = 2x - |2x+1| + |x|$

$\begin{array}{c} -\frac{1}{2} \quad 0 \\ 2x+1 \quad -1 \quad |x| \quad x-1 \end{array}$

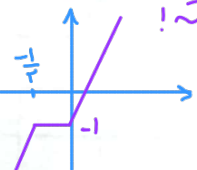
$x < -\frac{1}{2} \rightarrow 2x < -1 \rightarrow 2x+1 < 0$

$x > 0 \rightarrow 2x > 0 \rightarrow 2x+1 > 0$

$x > 0 \rightarrow 2x > 0 \rightarrow 2x+1 > 0$

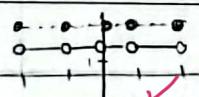


$x > 0 \rightarrow 2x > 0 \rightarrow 2x+1 > 0$



$R_y = \mathbb{R}$

الف) $y = [x] + [3-x]$

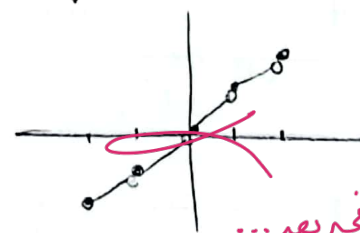


$R_y = \{1, 2, 3\}$

(1) (9)

ب) $y = 2x - [2]$

$\begin{cases} -2 \leq x < -1 \rightarrow 2x+2 \\ -1 \leq x < 0 \rightarrow 2x+1 \\ 0 \leq x < 1 \rightarrow 2x \\ 1 \leq x < 2 \rightarrow 2x-1 \\ x=2 \rightarrow 2x-2 \end{cases}$



متقطع

الف) $\sin^2 x - \sin x + 1$

$\sin x = 1 \rightarrow y = 1$

$\sin x = -1 \rightarrow y = 3$

$\sin x = \frac{1}{2} \rightarrow y = \frac{5}{4}$

$R = [\frac{5}{4}, 3]$

(2) (10)

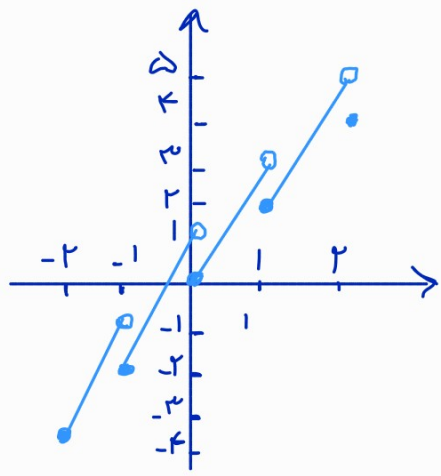
ب) $\sin^2 x + \sin^2 x + 1$

$\sin^2 x = 0 \rightarrow y = 1$

$R = [1, 3]$

$\sin^2 x = 1 \rightarrow y = 3$

$\sin^2 x = \frac{1}{2} \rightarrow y = \frac{3}{2}$



$$R_y = [-1, \infty)$$