

$$y = \frac{|x|+1}{|x|-1} \rightarrow y|x|-1 = |x|+1 \rightarrow |x|(y-1) = y+1 \rightarrow |x| = \frac{y+1}{y-1} \quad (1)$$

$$\frac{\frac{1}{y-1}}{\frac{1}{y-1}-1} \rightarrow (-\infty, -\frac{1}{2}] \cup (1, +\infty) = R_y$$

$$y = \frac{x^2+1}{|x|} \rightarrow \frac{x^2}{|x|} + \frac{1}{|x|} = \frac{|x|}{a} + \frac{1}{|x|} \rightarrow a > 0 \rightarrow a + \frac{1}{a} \geq 2 \rightarrow R_y = [2, +\infty)$$

$$y = \frac{x^2+x-1}{x+1} \rightarrow xy + y = x^2+x-1 \rightarrow x^2 + (\frac{1}{y}-1)x - y - 1 = 0 \quad (2)$$

$$(1-y)^2 - 4(-1-y) \geq 0 \rightarrow 1 - 2y + y^2 + 4 + 4y = y^2 + 2y + 5 = (y+1)^2 + 4 > 0$$

$$\frac{-4}{+1} \frac{-2}{-1} \rightarrow R_y = (-\infty, -4] \cup [-2, +\infty)$$

$$t \frac{x^2-x+1}{x^2-x+1} = \frac{t+1}{t+1} \quad x^2-x = (x-\frac{1}{2})^2 - \frac{1}{4} \quad \Delta K^0 \text{ زیر } R \text{ و } b \quad (3)$$

$$t \in [-\frac{1}{4}, +\infty) \rightarrow \frac{t+1}{t+1} = \frac{(t+1) - \frac{1}{4}}{t+1} = 1 - \frac{\frac{1}{4}}{t+1} \quad t = -\frac{1}{4} \rightarrow 1$$

$$R \in [0, 1)$$

$$a) \sqrt{-x^2+5x-1} \rightarrow -x^2+5x-1 \geq 0 \rightarrow x^2-5x+1 \leq 0 \rightarrow \Delta = 25-4 = 21 \rightarrow x = \frac{5 \pm \sqrt{21}}{2} \rightarrow y \min = 0, y \max = y_s \quad (4)$$

$$x_s = \frac{5+\sqrt{21}}{2} \rightarrow y_s = 3 \rightarrow -x^2+5x-1 \leq 3 \rightarrow \sqrt{-x^2+5x-1} \leq \sqrt{3} \rightarrow R = [0, \sqrt{3}]$$

$$b) y = \sqrt[3]{x^2-1} \rightarrow x_s = 1 \rightarrow y_s = -1 \rightarrow x^2-1 \geq -1 \rightarrow \sqrt[3]{x^2-1} \geq -1 \rightarrow R_y = [-1, +\infty)$$

$$a) y = \log(x^2-x+1) \rightarrow x^2-x+1 > 0 \rightarrow R_y = R \quad (5)$$

$$b) \log(\frac{x^2+1}{x}) \rightarrow \frac{x^2+1}{x} > 0 \rightarrow D_y = (0, +\infty)$$

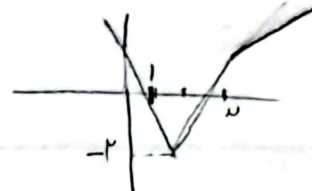
$$x + \frac{1}{x} \xrightarrow{a>0} x + \frac{1}{x} \geq 2 \rightarrow \min = 2 \rightarrow R_y = [\log 2, +\infty)$$

$$R_1 = [1, +\infty)$$

الف) $y = |x+1| - |x-3|$

$R = [-2, +\infty)$

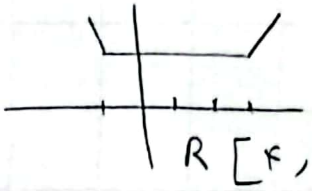
$\frac{-1 \quad +3}{-x-1 \quad x-3 \quad x+1}$



(6)

ب) $y = |x-3| + |x+1|$

$\frac{-1 \quad 3}{-x+3 \quad x \quad x+1}$



$R [3, +\infty)$

$|x+1| - |x-3| \leq 1$

$\frac{-1 \quad 3}{x+1 \quad x-3 \quad x+1}$

$x \leq -3$ I

$x < -1 \rightarrow x \leq -3$

$x > -1 \rightarrow x \leq -3$

$x > 0 \rightarrow x \leq -3$

$x \geq 3 \rightarrow x \leq -3$ III

$I \cap II \cap III \rightarrow (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$

(7)

$y = x - |x+1| + |x|$

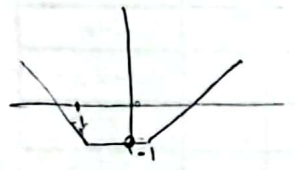
$\frac{-1 \quad 0}{x+1 \quad -1 \quad x-1}$

$x < -\frac{1}{3} \rightarrow x < -1 \rightarrow x+1 < 0$

$x < -1 \rightarrow x+1 < 0$

$x > 0 \rightarrow x > -1$

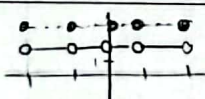
$x > 0 \rightarrow x > -1$



$\rightarrow [-1, +\infty)$

(8)

الف) $y = [x] + [3-x]$

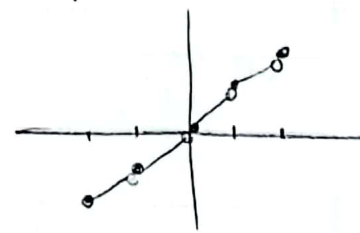


$R_y \{1, 3\}$

(9)

ب) $y = x - [x]$

$\begin{cases} -1 \leq x < 0 \rightarrow x+1 \\ 0 \leq x < 1 \rightarrow x \\ 1 \leq x < 2 \rightarrow x-1 \\ x=2 \rightarrow x-2 \end{cases}$



الف) $\sin^2 x - \sin x + 1$

$\rightarrow \sin x = 1 \rightarrow y = 1$

$R = [\frac{1}{2}, 3]$

(10)

$\frac{1}{2} \rightarrow y = 3$

$\frac{1}{2} \rightarrow y = \frac{1}{2}$

ب) $\sin^2 x + \sin^2 x + 1$

$\rightarrow \sin^2 x = 0 \rightarrow y = 1$

$\sim R [1, 3]$

$\frac{1}{2} \rightarrow y = 3$

$\frac{1}{2} \rightarrow y = \frac{1}{2}$

$\frac{1}{2} \rightarrow y = \frac{1}{2}$