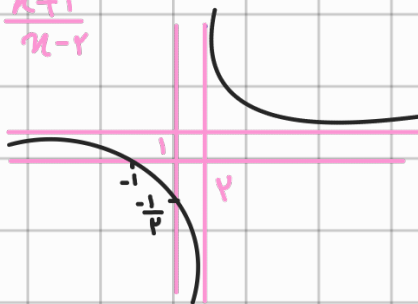


$$y = \frac{|x|+1}{|x|-2}$$

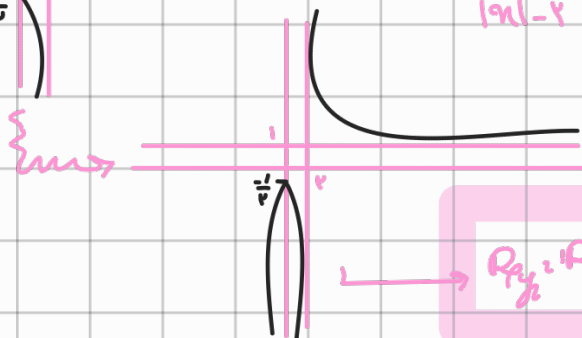
$$y = \frac{x+1}{x-2}$$



فرض $x > 2$

$$y = \frac{|x|+1}{|x|-2}$$

فرض $x < -2$



$$D_{f_2} = \mathbb{R} - (-\frac{1}{2}, 1]$$

$$y = \frac{x^2+1}{|x|} \rightsquigarrow x > 0 \rightsquigarrow \frac{x^2+1}{x} = x + \frac{1}{x} \rightsquigarrow x + \frac{1}{x} \geq 2 \quad \textcircled{1}$$

فرض $x < 0$

$$\rightsquigarrow x < 0 \rightsquigarrow \frac{x^2+1}{-x} = -x + \frac{1}{-x} \rightsquigarrow -x + \frac{1}{-x} \leq -2$$

$$\sum_{x=1}^{\infty} x + \frac{1}{x} \geq 2 \quad \textcircled{1}$$

$$\textcircled{1}, \textcircled{2} \rightsquigarrow D_{f_2} = [2, +\infty)$$

$$y = \frac{x^2 + 4x - 2}{x + 3} \rightsquigarrow y_2 x + 3y = x^2 + 4x - 2 \rightsquigarrow x^2 + x(4 - y) - 2 - 3y_2$$

فرض

$$\Delta = (4 - y_2)^2 - 4(-2 - 3y_2) = y_2^2 - 8y_2 + 8 + 12 + 12y_2 = y_2^2 + 4y_2 + 20 = (y_2 + 2)(y_2 + 10)$$

$$x = \frac{y - 4 \pm \sqrt{(y+2)(y+10)}}{2} \rightsquigarrow \begin{array}{c} -4 \quad -2 \\ + \quad - \end{array} \rightsquigarrow D_{f_2} = (-\infty, -4] \cup [-2, +\infty)$$

$$y = \frac{x^2 - x + \frac{1}{x}}{x^2 - x + 1} \rightsquigarrow$$

$$x^2 - x + 1 \neq 0$$

$$1 - x \times |x| = -x \rightsquigarrow$$

$$D_{f_2} = \mathbb{R}$$

$$R_y \rightarrow yx^2 - yx + y = x^2 - x + \frac{1}{y} \rightarrow x^2(1-y) + x(y-1) + \frac{1}{y} = 0$$

$$\Delta = (y-1)^2 - 4 \times \left(\frac{1}{y}\right) \times (1-y) = y^2 - 2y + 1 - 4y^2 + 4y = -3y^2 + 2y$$

$\xrightarrow{\text{Factor}} -y(3y-2)$

$$x = \frac{1-y \pm \sqrt{y(-3y+2)}}{y-1} \xrightarrow{\Delta \geq 0} \begin{array}{c} 0 \quad 1 \\ - \quad + \quad - \\ 0 \quad 0 \end{array} \left\{ \begin{array}{l} \text{موجب} \\ \text{منفي} \end{array} \right\} \Rightarrow R_y = [0, 1)$$

$\hookrightarrow y-1 \neq 0 \rightarrow y \neq 1$

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$$y = \sqrt{-x^2 + 4x - 1} \Rightarrow y^2 = -x^2 + 4x - 1 \Rightarrow -x^2 + 4x - 1 - y^2 = 0$$

$$\Delta = 16 - 4 \times (-1) \times (-1 - y^2) \Rightarrow \Delta = 12 - 4y^2$$

$\xrightarrow{\text{Factor}} -4(3 - y^2)$

$$x = \frac{-4 \pm \sqrt{12 - 4y^2}}{-2} \xrightarrow{\Delta \geq 0} \begin{array}{c} -\sqrt{3} \quad \sqrt{3} \\ - \quad + \quad - \\ 0 \quad 0 \end{array} \left\{ \begin{array}{l} \text{موجب} \\ \text{منفي} \end{array} \right\} \Rightarrow R_y = [0, \sqrt{3}]$$

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$$y = \sqrt[3]{4x^2 - 12x} \Rightarrow y^3 = 4x^2 - 12x \Rightarrow 4x^2 - 12x - y^3 = 0$$

$$\Delta = 144 - 4 \times 4 \times (-y^3) = 144 + 16y^3 = 16(9 + y^3)$$

$$x = \frac{3 \pm \sqrt{9 + y^3}}{2} \xrightarrow{\Delta \geq 0} \begin{array}{c} - \quad + \\ - \quad 0 \end{array} \Rightarrow R_y = [-2, +\infty)$$

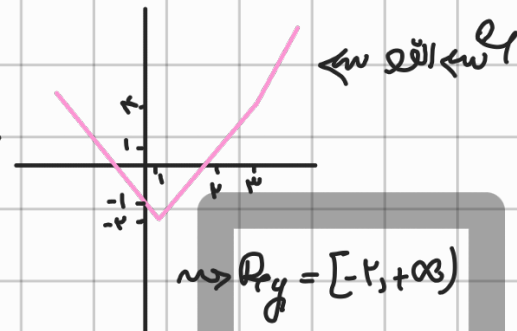
$$y_2 = \log_{\frac{1}{2}} x^{\frac{1}{2}} - x + 1 \Rightarrow \text{دالة متزايدة في } \mathbb{R}^+ \Rightarrow P_{y_2} = (-\infty, +\infty)$$

$$\Rightarrow P_{y_2} = \mathbb{R}$$

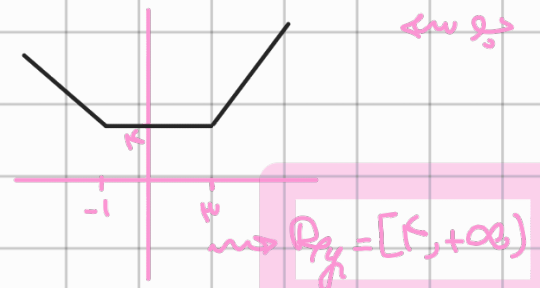
$$y_2 = \log_{\frac{1}{2}} \frac{x^{\frac{1}{2}} + 1}{x} \xrightarrow{\log \frac{1}{2}} \frac{x^{\frac{1}{2}} + 1}{x} > 0 \Rightarrow x > 0 \Rightarrow x + \frac{1}{x} \geq 2 \quad (1)$$

$$\xrightarrow{(1)} \log_{\frac{1}{2}} \frac{x^{\frac{1}{2}} + 1}{x} \geq \log_{\frac{1}{2}} 2 \Rightarrow y_2 \geq 1 \Rightarrow P_{y_2} = [1, +\infty)$$

$$y_2 = |2x - 4| - |x - 3| \Rightarrow \begin{matrix} x \geq 4 & 3x - 7 \\ -x - 1 & x < 4 \end{matrix}$$



$$y_2 = |x - 3| + |x + 1| \Rightarrow \begin{matrix} x \geq 3 & 2x - 2 \\ -1 \leq x < 3 & -2 \\ x < -1 & -2x + 4 \end{matrix}$$



$$|x - 2| - |x + 4| \leq 1$$

$$\begin{cases} -x - 6 & x \geq 2 \\ -3x & -4 \leq x < 2 \\ x + 6 & x < -4 \end{cases}$$



$$y_2 = \psi n - |\psi n + 1| + |n| \rightsquigarrow \psi n - 1 \quad n \geq 0$$

$$\begin{matrix} -1 & -\frac{1}{\psi} \leq n < 0 \\ \psi n + 1 & n < -\frac{1}{\psi} \end{matrix}$$

$\rightsquigarrow R_{y_2} = \mathbb{R}$

$$y_2 = [n] + [\psi - n] \rightsquigarrow$$

$R_{y_2} = \{\psi, \psi\}$

$$y_2 = \psi n - [n] \rightsquigarrow$$

$R_{y_2} = [-\psi, \omega)$

$$y_2 = \sin^{\psi} n - \sin n + 1 \rightsquigarrow y_2 = 1 - 1 + 1 \rightsquigarrow y_2 = 1$$

$$\rightsquigarrow y_2 = 1 + 1 + 1 \rightsquigarrow y_2 = 3$$

$$\rightsquigarrow y_2 = \frac{1}{\psi} - \frac{1}{\psi} + 1 = \frac{2}{\psi}$$

$\rightsquigarrow R_{y_2} = [\frac{\psi}{\psi}, \psi]$

$$y_2 = \sum_{i=1}^n x_i + \sum_{i=1}^n x_{i+1}$$

$\xrightarrow{\sum x_i = 1} y = 1 + 1 + 1 \rightarrow y = 3$
 $\xrightarrow{\sum x_i = 0} y = 0 + 0 + 1 \rightarrow y = 1$
 $\xrightarrow{\sum x_i = -\frac{1}{2}} y = -\frac{1}{2} - \frac{1}{2} + 1 = 0$
 $\xrightarrow{\sum x_i \neq -\frac{1}{2}} y \neq 0$

$\rightarrow \text{Range} = [1, 3]$

$\leftarrow$ در این حالت y از 0 تا 3 متغیر می‌شود