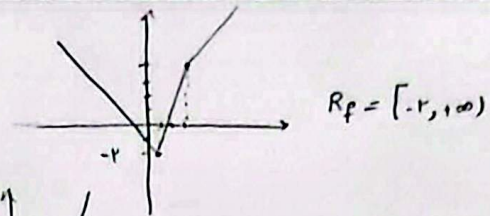


الف) $y = \frac{ x +1}{ x -2} \Rightarrow y x - 2y = x +1 \rightarrow y x - x = 1+2y = x (y-1)$ $\Rightarrow x = \frac{1+2y}{y-1} \rightarrow x \geq 0 \Rightarrow \frac{1+2y}{y-1} \geq 0 \rightarrow \frac{1}{y-1} \geq -2 \rightarrow R_f = \mathbb{R} - (-\frac{1}{2}, 1]$ ب) $y = \frac{x^2+1}{ x } \rightarrow y = x + \frac{1}{ x } \rightarrow x > 0 \Rightarrow R_f = [1, +\infty)$	۱
$y = \frac{x^2+2x-2}{x+2} \Rightarrow yx + 2y = x^2+2x-2 \rightarrow x^2 + (2-y)x - (2+2y) = 0$ $\rightarrow \Delta = (2-y)^2 - 4(1)(-(2+2y)) = 4 - 4y + y^2 + 8y + 8 = y^2 + 4y + 12 = (y+2)(y+4)$ $\rightarrow x = \frac{y-2 \pm \sqrt{y^2+4y+12}}{2} \Rightarrow y^2 + 4y + 12 \geq 0 \Rightarrow (y+2)(y+4) \geq 0$ $\rightarrow \frac{-4}{2} \leq y \leq \frac{-2}{2} \Rightarrow R_f = \mathbb{R} - (-4, -2)$	۲
$D_f: x^2 - x + 1 \neq 0 \rightarrow \Delta = 1 - 4 = -3 \rightarrow$ همیشه مثبت $\rightarrow D_f = \mathbb{R}$ $y = \frac{x^2-x+\frac{1}{x}}{x^2-x+1} \rightarrow yx^2 - yx + y = x^2 - x + \frac{1}{x} \rightarrow x^2(1-y) + x(y-1) + \frac{1}{x} - y$ $\Delta = (y-1)^2 - 4(\frac{1}{x}-y)(1-y) = (y-1)^2 + (4y-1)(1-y) = y^2 - 2y + 1 + 4y - 1 - 4y^2 = -3y^2 + 4y$ $= -3y^2 + 4y \geq 0 \rightarrow y(4-3y) \geq 0 \rightarrow 0 \leq y \leq \frac{4}{3}$ $\frac{0}{-3} \leq y \leq \frac{4}{3}$ $\frac{0}{-3} \leq y \leq \frac{4}{3} \rightarrow \frac{1}{x} = 1 \rightarrow x = 1 \rightarrow$ غلط $\Rightarrow y \neq 1 \rightarrow R_f = [0, 1)$	۳
الف) $y = \sqrt{-x^2+4x-1} \rightarrow$ برد: $-x^2+4x-1 \geq 0 \Rightarrow x^2-4x+1 \leq 0 \Rightarrow \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-4) \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$ $\rightarrow R = (-\infty, 2-\sqrt{3}] \cup [2+\sqrt{3}, +\infty) \rightarrow \sqrt{(-\infty, 2-\sqrt{3}] \cup [2+\sqrt{3}, +\infty)} = [0, \sqrt{3}] \Rightarrow R_f = [0, \sqrt{3}]$ ب) چنانچه دهم نزد است پس محدودیتی روی عبارت زیر آن وجود ندارد پس هر مقدار از عبارت زیر آن قابل قبول است و داریم: $y = \frac{x^2-2x}{x^2-2x+1} \Rightarrow \frac{-b}{2a} = \frac{2}{2} = 1 \rightarrow 1-2y = -1 \rightarrow R = [-1, +\infty)$ $\rightarrow \sqrt{[-1, +\infty)} = [-1, +\infty) = R_f$	۴
الف) برد: $x^2 - x + 1 \rightarrow$ بزرگترین دهم نزد $\rightarrow R = \mathbb{R}$ $\rightarrow y = \log(x^2 - x + 1) = \log \mathbb{R} \rightarrow \log(0, +\infty) \rightarrow R_f = \mathbb{R}$ ب) برد: $\frac{x^2+1}{x} = x + \frac{1}{x} \Rightarrow R = (-\infty, -2] \cup [2, +\infty)$ $\rightarrow y = \log_r(\frac{x^2+1}{x}) = \log_r(-\infty, -2] \cup [2, +\infty) = \log_r[2, +\infty) \Rightarrow R_f = [1, +\infty)$	۵

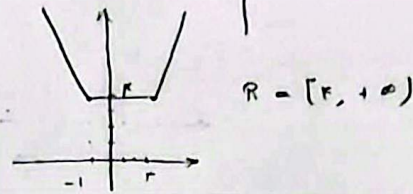
$$\text{الف) } \begin{cases} x \geq 2 \rightarrow x - 2 - x + 1 = x + 1 \\ -1 \leq x \leq 2 \rightarrow x - 2 + x - 1 = 2x - 3 \\ x \leq -1 \rightarrow x - 2 + x - 1 = -x - 1 \end{cases}$$

→



6

ب) $x \geq -1$: $x - 2 + x - 1 = 2x - 3$



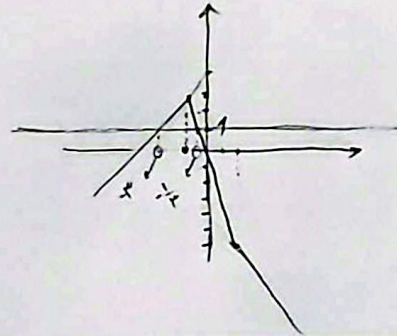
$$y = |x - 2| - |2x + 2|$$

$$\begin{cases} x \geq 2 \rightarrow x - 2 - 2x - 2 = -x - 4 \\ -1 \leq x \leq 2 \rightarrow x - 2 - 2x - 2 = -x - 4 \\ x \leq -1 \rightarrow x - 2 + 2x + 2 = 3x \end{cases}$$

$$-x - 4 = 1 \rightarrow x = -5$$

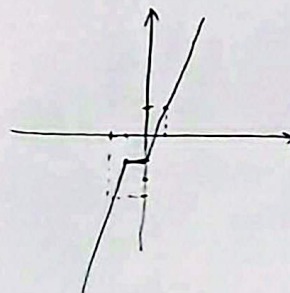
$$3x = 1 \rightarrow x = \frac{1}{3}$$

$$\Rightarrow \text{مجموعه جواب} = \mathbb{R} - (-5, \frac{1}{3})$$



7

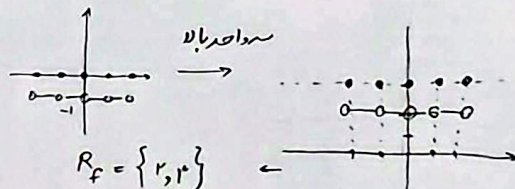
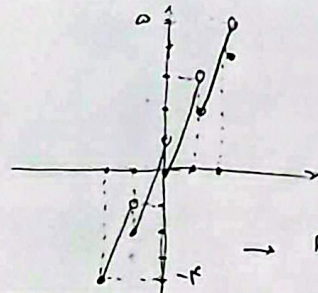
$$\begin{cases} x \geq 0 \rightarrow x - 2x - 1 + x = x - 1 \\ -\frac{1}{2} \leq x \leq 0 \rightarrow x - 2x - 1 - x = -1 \\ x \leq -\frac{1}{2} \rightarrow x - 2x - 1 - x = -x - 1 \end{cases}$$



8

الف) $y = [x] + [-x] + 2 \rightarrow$ نمودار $[x] + [-x]$:

ب) $[-2, -1) \rightarrow x + 2$
 $[-1, 0) \rightarrow x + 1$
 $[0, 1) \rightarrow x$
 $[1, 2) \rightarrow x - 1$
 $\{2\} \rightarrow x - 2$



9

الف) $y = \sin^2 x - \sin x + 1 : -1 \leq \sin x \leq 1 \rightarrow \sin x = 1 : y = 1$
 $\sin x = -1 : y = 3 \rightarrow R_f = [\frac{1}{2}, 3]$
 $-\frac{b}{2a} = \frac{1}{2} \rightarrow \sin x = \frac{1}{2} \rightarrow y = \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}$

10

ب) $y = \sin^2 x + \sin x + 1 : 0 \leq \sin x \leq 1 \rightarrow \sin x = 1 \rightarrow y = 3$
 $\sin x = 0 \rightarrow y = 1 \rightarrow R_f = [1, 3]$
 $-\frac{b}{2a} = -\frac{1}{2} \rightarrow \sin x \neq -\frac{1}{2}$