

۱)

الف) $y = \frac{\ln|t|+1}{\ln|t|-1} \rightarrow y(\ln|t|-1) = \ln|t|+1 \rightarrow \ln|t|(y-1) = y+1 \rightarrow \ln|t| = \frac{y+1}{y-1}$

$\frac{y+1}{y-1} > 0 \rightarrow \frac{1}{1-\frac{1}{y}} > 0 \rightarrow R_f = (-\infty, -1] \cup (1, +\infty)$

ب) $y = \frac{x^2+1}{x^2} \rightarrow y(x^2+1) = x^2+1 \rightarrow R_f = [1, +\infty)$

۲)

$y = \frac{x^2+x-1}{x+1}$

$yx + y - x^2 - x + 1 = x^2 + x - 1 \rightarrow x^2 + (1-y)x - 2 = 0$

$\Delta = \frac{b^2 - 4ac}{4a^2} \geq 0 \rightarrow (1-y)^2 - 4(-2) \geq 0 \rightarrow 1 - 2y + y^2 + 8 \geq 0 \rightarrow y^2 - 2y + 9 \geq 0$

$y^2 - 2y + 9 \geq 0 \rightarrow (y-1)^2 + 8 \geq 0 \rightarrow \frac{y-1}{1-\frac{1}{y}} \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

۳)

$f(x) = \frac{x^2-x+1}{x^2-x+1}$

$\Delta = 1 - 4 = -3 < 0 \rightarrow D_f = \mathbb{R}$

$y \neq 1 \rightarrow x = \frac{-(1-y) \pm \sqrt{\Delta}}{2(y-1)}$

$\Delta \geq 0 \rightarrow (1-y)^2 - 4(y-1)(y-\frac{1}{y}) \geq 0 \rightarrow \frac{y-1}{-1+\frac{1}{y}} \rightarrow R_f = [0, 1)$

۴)

الف) $y = \sqrt{x^2+x-1}$

$\frac{b}{2a} = \frac{-1}{2} = -\frac{1}{2}$

$\rightarrow R_f = \sqrt{(-\infty, 1]} = [-1, 1]$

ب) $y = \sqrt{x^2-1}$

$\frac{b}{2a} = \frac{0}{2} = 0$

$\rightarrow R_f = \sqrt{[1, +\infty)} = [1, +\infty)$

۵)

الف) $y = f(x^2-x+1)$

$R_f = f(\mathbb{R}) = (-\infty, +\infty)$

ب) $y = \log(\frac{x^2+1}{x})$

$\rightarrow R_f = \log([1, +\infty)) = [0, +\infty)$

$\frac{x^2+1}{x} > 0 \rightarrow x > 0 \rightarrow \frac{x^2+1}{x} \in [1, +\infty)$

4)

a) $y = |x-1| - |x-3|$

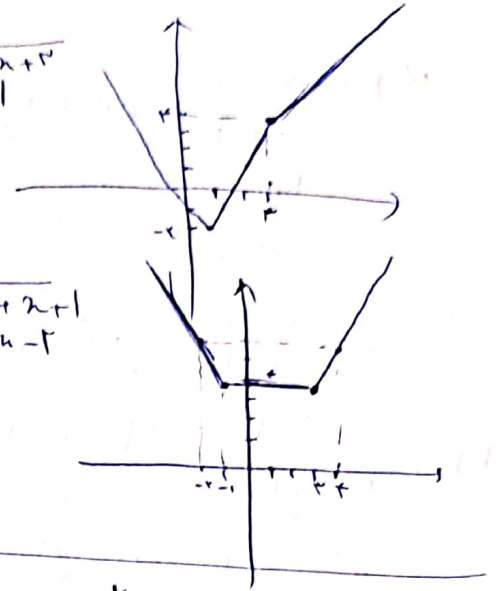
$x-1 > 3 \Rightarrow x > 4$	$x-1 < 3 \Rightarrow x < 4$	$x-1 < 3 \Rightarrow x < 4$
$x-1 > 3 \Rightarrow x > 4$	$x-1 < 3 \Rightarrow x < 4$	$x-1 < 3 \Rightarrow x < 4$
$x-1 > 3 \Rightarrow x > 4$	$x-1 < 3 \Rightarrow x < 4$	$x-1 < 3 \Rightarrow x < 4$

$y \in [-2, +\infty)$

b) $y = |x-1| + |x+1|$

$x-1 > -1 \Rightarrow x > 0$	$x-1 < -1 \Rightarrow x < 0$	$x-1 < -1 \Rightarrow x < 0$
$x-1 > -1 \Rightarrow x > 0$	$x-1 < -1 \Rightarrow x < 0$	$x-1 < -1 \Rightarrow x < 0$
$x-1 > -1 \Rightarrow x > 0$	$x-1 < -1 \Rightarrow x < 0$	$x-1 < -1 \Rightarrow x < 0$

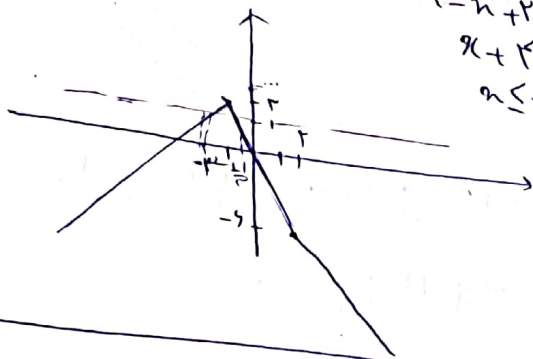
$y \in [2, +\infty)$



v) $|x-1| - |x+1| \leq 1$

$x-1 > -1 \Rightarrow x > 0$	$x-1 < -1 \Rightarrow x < 0$	$x-1 < -1 \Rightarrow x < 0$
$x-1 > -1 \Rightarrow x > 0$	$x-1 < -1 \Rightarrow x < 0$	$x-1 < -1 \Rightarrow x < 0$
$x-1 > -1 \Rightarrow x > 0$	$x-1 < -1 \Rightarrow x < 0$	$x-1 < -1 \Rightarrow x < 0$

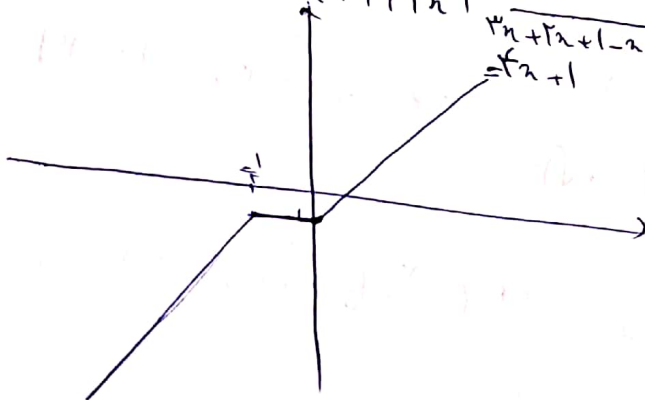
$x \in (-\infty, -1] \cup [\frac{1}{2}, +\infty)$



1) $y = x - |x+1| + |x|$

$x > 0$	$x < 0$	$x < 0$
$x > 0$	$x < 0$	$x < 0$
$x > 0$	$x < 0$	$x < 0$

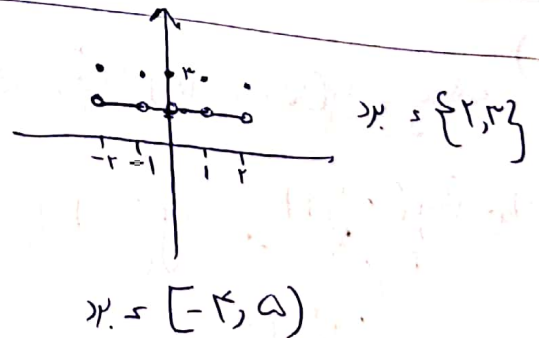
$y \in [0, 1]$



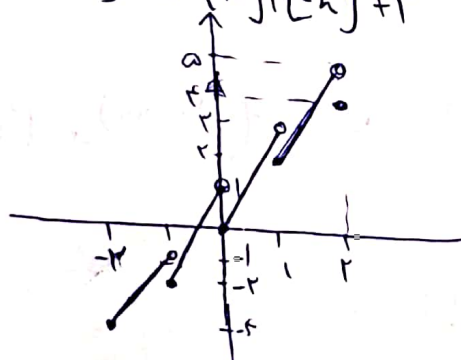
9)

a) $y = [x] + [x-1] = [x] + [-x] + 1$

$-1 < x \leq 0$



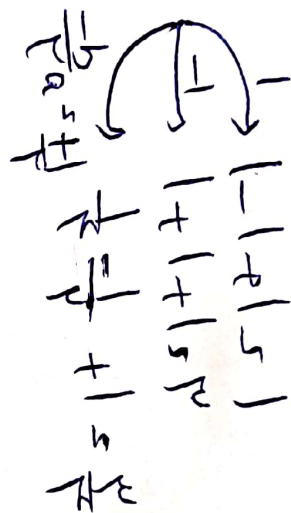
b) $y = x - [x]$



$y \in [0, 1)$

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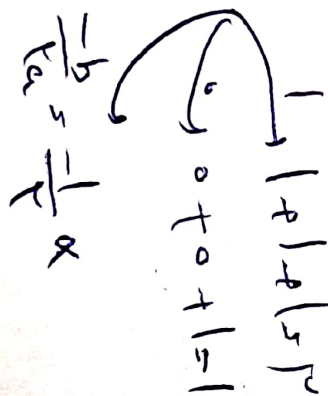
$$g_{\text{sing}} - \text{sing} + 1$$



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25
[14]
[5]

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Disinfectant



27, 28, 29