

دالة دالة

تطبيق

المعادلة

$$y = \frac{|x|+1}{|x|-2} \Rightarrow y|x|-2y = |x|+1$$

$$|x|(y-1) = 2y+1$$

$$\Rightarrow |x| = \frac{2y+1}{y-1}$$

$$R_f = (-\infty, -\frac{1}{2}] \cup (1, +\infty)$$

$$y = \frac{x^2+1}{|x|} \Rightarrow y|x| = x^2+1$$

$$\Rightarrow x^2 - y|x| + 1 = 0$$

$$\Delta = y^2 - 4(1)(1) = y^2 - 4$$

$$y \geq 2 \text{ or } y \leq -2$$

$$x_{1,2} = \frac{y \pm \sqrt{y^2-4}}{2}$$

$$R_f = [2, +\infty) \cup (-\infty, -2]$$

$$y = \frac{x^2+2x-1}{x+1} \Rightarrow xy + 1^2y = x^2 + 2x - 1 \Rightarrow x^2 + (1-y)x - 1 - 1^2y = 0$$

$$\Delta = (1-y)^2 - 4(1)(-1-y) = 1 - 2y + y^2 + 4 + 4y = y^2 + 2y + 5 = (y+1)^2 + 4$$

$$x_{1,2} = \frac{-1 \pm \sqrt{(y+1)^2 + 4}}{2}$$

$$R_f = (-\infty, -4] \cup [-2, +\infty)$$

$$f(x) = \frac{x^2 - x + \frac{1}{2}}{x^2 - x + 1}$$

$$D_f = \mathbb{R} \Rightarrow \frac{(x - \frac{1}{2})^2}{(x - \frac{1}{2})^2 + \frac{3}{4}}$$

$$R_f = [0, 1]$$

$$y = \sqrt{-x^2 + 8x - 1}$$

$$\Delta = 14 - 4(-1)(-1) = 12$$

$$-\frac{\Delta}{4a} = \frac{-12}{-2} = 6$$

$$R_f = [0, \sqrt{12}]$$

$$y = \sqrt{x^2 - 12x}$$

$$\Delta = 48$$

$$-\frac{\Delta}{4a} = \frac{-48}{2} = -24$$

$$R_f = [-24, +\infty)$$

$$y = \log(x^2 - x + 1)$$

$$x^2 - x + 1 > 0$$

$$R_f = \mathbb{R}$$

$$y = \log_r\left(\frac{x^2+1}{x}\right)$$

$$\frac{x^2+1}{x} > 0$$

$$x > 0$$

$$R_f = (1, +\infty)$$

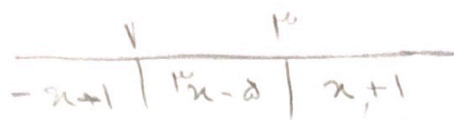
$$x + \frac{1}{x} \Rightarrow \frac{x^2+1}{x}$$

$$R_f = (1, +\infty)$$

$$a) f = |rx - r| - |x - r|$$

$$g = |x - r| + |x + 1|$$

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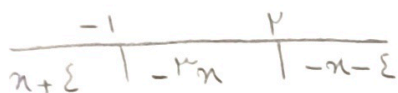


$$R_f = [r, +\infty)$$



$$R_g = [r, +\infty)$$

$$|x - r| - |rx + r| \leq 1$$



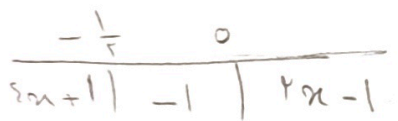
$$-x - \varepsilon \leq +1 \rightarrow x \geq -\omega$$

$$R_f = (-\infty, -r] \cup [-\frac{1}{r}, +\infty)$$

$$-rx \leq +1 \rightarrow x \geq -\frac{1}{r}$$

$$x + \varepsilon \leq +1 \rightarrow x \leq -r$$

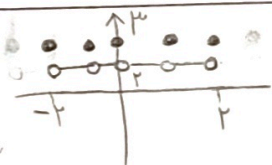
$$g = rx - |rx + 1| + |x|$$



$$R_g = \mathbb{R}$$

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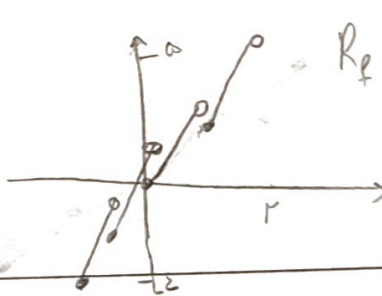
$$y = [x] + [-x] + r$$



$$R_f = \{r, r\}$$

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$$y = rx - [x] \rightarrow \begin{aligned} -r \leq x < -1 &\rightarrow rx + r \\ -1 \leq x < 0 &\rightarrow rx + 1 \\ 0 \leq x < 1 &\rightarrow rx \\ 1 \leq x < r &\rightarrow rx - 1 \\ x = r &\rightarrow rx - r \end{aligned}$$



$$R_f = [-\varepsilon, \omega)$$

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$$a) \sin^r x - \sin x + 1$$

$$R_f = \left[\frac{r}{\varepsilon}, r\right]$$

$$\sin x = 1 \rightarrow y = 1$$

$$\sin x = -1 \rightarrow y = r$$

$$\sin x = \frac{1}{r} \rightarrow y = \frac{1}{\varepsilon} - \frac{1}{r} + 1 = \frac{r}{\varepsilon}$$

$$b) \sin^r x + \sin^r + 1$$

$$R_f = [1, r]$$

$$\sin^r x = 0 \rightarrow y = 1$$

$$\sin^r x = 1 \rightarrow y = r$$

$$\sin^r x = \frac{1}{r} \rightarrow y = \frac{1}{\varepsilon} - \frac{1}{r} + 1 = \frac{r}{\varepsilon}$$