

201) $y, \frac{|a|+1}{|a|-1} \rightarrow y = \frac{-y-1}{y-1} = \frac{y+1}{y-1}$ -1

$|a| = \frac{y+1}{y-1} \geq 0 \rightarrow \frac{-\frac{1}{y}+1}{1-\frac{1}{y}} + \rightarrow R_f = \mathbb{R} - (-\frac{1}{y}, 1]$ (2)

$|a| + \frac{1}{|a|} = |a| \geq 0$

$R_f = [1, +\infty)$

$y, \frac{a^2+1}{a+1} \rightarrow ay+1y = a^2+1a-1 \rightarrow a^2+a(y-1)-1-y=0$ -2

$a = \frac{-(y-1) \pm \sqrt{(y-1)^2 + 4(y+1)}}{2} = \frac{-(y-1) \pm \sqrt{y^2+4y+5}}{2}$ (2)

$y^2+4y+1 \geq 0 \rightarrow (y+2)(y+1) \geq 0 \rightarrow \frac{-y-2}{1-\frac{1}{y}} + \rightarrow R_f = \mathbb{R} - (y-2)$

$f(x) = \frac{a^2-a+1}{a^2-a+1} \rightarrow a^2-a+1 = 0 \rightarrow \Delta < 0 \rightarrow a^2-a+1 \neq 0$ -2

$D_f = \mathbb{R} \rightarrow y, a^2-a+1 \geq 0$ (2)

$a^2(1-y) + a(y-1) + \frac{1}{y} - y \rightarrow a = \frac{1-y \pm \sqrt{(y-1)^2 - 4(\frac{1}{y}-y)}}{2(1-y)}$

$-y^2+4y \geq 0 \rightarrow \frac{0}{1-\frac{1}{y}} + \rightarrow R_f = [0, 1]$

21) $y, \sqrt{-a^2+1a-1} \rightarrow \frac{-b}{2a} = \frac{-1}{-2} = \frac{1}{2}, y = \frac{1}{2}$ -2

$\Delta = 1-4 = -3 < 0 \rightarrow R_f = (-\infty, \frac{1}{2}]$ (2)

$\sqrt{(-\infty, \frac{1}{2}]} = [-\frac{1}{2}, \frac{1}{2}] = R_f$

$y, \sqrt{a^2-1a} \rightarrow \frac{-b}{2a} = \frac{1}{2} \rightarrow y = -\frac{1}{2}$

$R_f = [-\frac{1}{2}, +\infty) = \sqrt{[-1, +\infty)}$

$= [-1, +\infty) = R_f$

22) $y = \log(a^2-a+1)$ -2

$\log(\mathbb{R}) = \log((-\infty, +\infty)) = (-\infty, +\infty) = \mathbb{R}$ (2)

$R_f = \mathbb{R}$

$y = \log(\frac{a^2+1}{a}) \rightarrow \log(a + \frac{1}{a})$

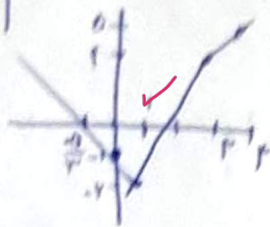
$a + \frac{1}{a} \rightarrow D_f = (-\infty, -1] \cup [1, +\infty)$

$\log([1, +\infty)) = \log_{\frac{1}{e}} \rightarrow \log_{\frac{1}{e}} = 1 \rightarrow [1, +\infty) = R_f$

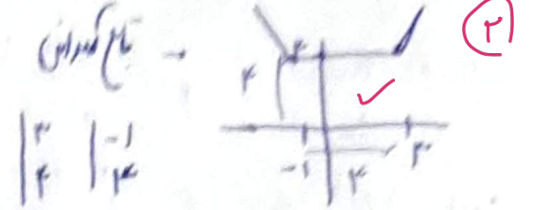
$$a) y = |x-1| - |x-2|$$

$$\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$R_f = [-1, +\infty)$$



$$\rightarrow y = |x-1| + |x+1|$$



$$R_f = [1, +\infty)$$

$$|x-1| - |x+1| \leq 1$$

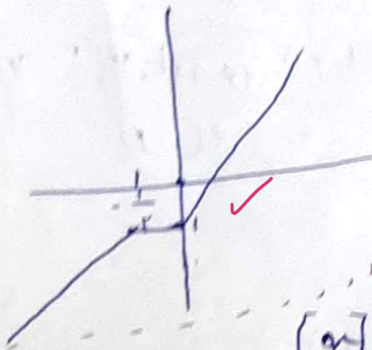
$$\begin{array}{c|c|c} -1 & 1 & \\ \hline x-1+x+1 \leq 1 & x-1-x-1 \leq 1 & x-1-x-1 \leq 1 \\ x \leq -1 & x \geq -\frac{1}{2} & x \geq 0 \end{array}$$

$$\textcircled{1} \cap \textcircled{2}, \cup \textcircled{3} = [-\frac{1}{2}, +\infty) \cup (-\infty, -1]$$

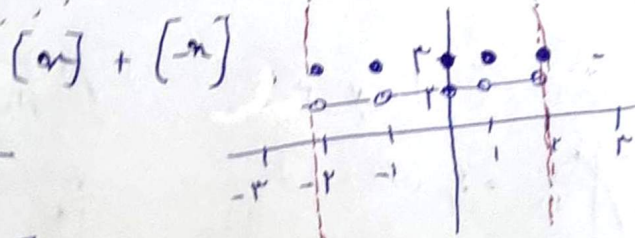
$$y = |x-1| - |x+1| + |x|$$

$$\begin{array}{c|c|c} -1 & 0 & \\ \hline x-1-x-1-x & x-1-x-1-x & x-1-x-1-x \\ x-1-x-1-x & -1 & x-1-x-1-x \\ x-1-x-1-x & -1 & x-1-x-1-x \end{array}$$

$$\rightarrow R_f = \mathbb{R}$$

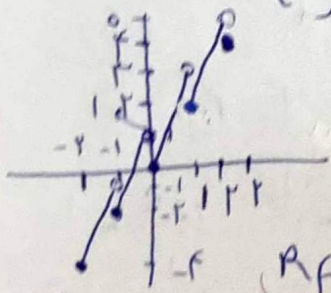


$$a) [x] + [2-x] = y \rightarrow [x] + [-x] + 2 = y$$



$$R_y = \{2\}$$

$$\rightarrow y = |x-1| - [x]$$



$$\begin{array}{l} -1 \leq x < 0 \rightarrow [x] = -1 \rightarrow |x-1| - (-1) \\ 0 \leq x < 1 \rightarrow [x] = 0 \rightarrow |x-1| - 0 \\ 1 \leq x < 2 \rightarrow [x] = 1 \rightarrow |x-1| - 1 \\ x = 2 \rightarrow [x] = 2 \rightarrow |x-1| - 2 \end{array}$$

$$R_f = [-1, 0)$$

$$a) y = \sin^2 x - \sin x + 1$$

$$R_f = [\frac{3}{4}, 2]$$

$$\begin{array}{l} \max = 1 \rightarrow 1 \\ \min = -1 \rightarrow 2 \end{array}$$

$$\rightarrow y = \sin^2 x + \sin x + 1$$

$$R_f = [1, 2]$$