

روز دهم دفتر کا سه شنبه

کالیف (ماده پنج ۱)

فایان میرزپور

$$\text{الف) } y = \frac{|x|+1}{|x|-2} \Rightarrow y|x| - 2y = |x| + 1 \Rightarrow y|x| - |x| = 1 + 2y$$

$$(y-1)|x| = 1+2y \Rightarrow |x| = \frac{1+2y}{y-1}$$

نسبت مثبت (مادی) نامنفی است

$$\begin{array}{c} -\frac{1}{2} \quad 1 \\ + \quad - \quad + \end{array}$$

$$R_y = (-\infty, -\frac{1}{2}] \cup (1, +\infty)$$

$$\text{ب) } y = \frac{x^2+1}{|x|} \Rightarrow y = |x| + \frac{1}{|x|} \Rightarrow R_y = [2, +\infty)$$

$$y = \frac{x^2+2x-2}{x+3} \Rightarrow y(x+3) = x^2+2x-2 \Rightarrow x^2+2x-yx-2-2y = 0$$

$$\Rightarrow x^2 - (y-2)x - 2-2y = 0 \Rightarrow x = \frac{y-2 \pm \sqrt{(y-2)^2 + 4(2+y)}}{2} \Rightarrow y^2+4y+12 \geq 0 \Rightarrow (y+2)(y+6) \geq 0$$

$$\Delta = (y-2)^2 - 4(-2-2y) = y^2-4y+4+8+8y = y^2+4y+12$$

$$\Rightarrow R_y = (-\infty, -6] \cup [-2, +\infty)$$

$$f(x) = \frac{x^2-x+\frac{1}{x}}{x^2-x+1} \Rightarrow f(x) = \frac{(x-\frac{1}{x})^2}{(x-\frac{1}{x})^2+\frac{4}{x}}$$

$$D_f = \mathbb{R} \quad \text{مخرج هیچگاه صفر نمی شود}$$

$$(x-\frac{1}{x})^2 \geq 0 \Rightarrow y = 1 \quad \text{(حدی نیست آدمیم)}$$

$$(x-\frac{1}{x})^2 = 0 \Rightarrow y = 0$$

$$R_{f(x)} = [0, 1]$$

$$\text{الف) } y = \sqrt{-x^2+4x-1}$$

هسته Max دارد : برد هسته
(a < 0)

$$\begin{array}{l} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ -4+4-1 = -1 \end{array}$$

$$\Rightarrow \left| \begin{array}{c} 2 \\ 3 \end{array} \right| \Rightarrow R = (-\infty, 3] \Rightarrow \sqrt{(-\infty, 3]} \Rightarrow R_y = [0, \sqrt{3}]$$

$$\text{ب) } y = \sqrt[3]{2x^2-4x}$$

هسته Min دارد : برد هسته
(a > 0)

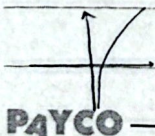
$$\begin{array}{l} -\frac{b}{2a} = \frac{4}{2} = 2 \\ 2-4 = -2 \end{array}$$

$$\Rightarrow \left| \begin{array}{c} 2 \\ -2 \end{array} \right| \Rightarrow R = [-2, +\infty) \Rightarrow \sqrt[3]{[-2, +\infty)} \Rightarrow R_y = [-\sqrt[3]{2}, +\infty)$$

$$\text{ان) } y = \log(x^2-x+1)$$

(هسته فیکه برای مرتبه فرد است) R : برد هسته

$$\Rightarrow \log \mathbb{R} \Rightarrow \log (0, +\infty) \Rightarrow R_y = (-\infty, +\infty)$$



ب) $y = \log_2 \left(\frac{x^2+1}{x} \right)$ $\frac{x^2+1}{x} = x + \frac{1}{x} \Rightarrow R = (-\infty, -2] \cup [2, +\infty)$
 همة

$\Rightarrow \log_2 [2, +\infty) \Rightarrow R_y = [1, +\infty)$ جواب

5 الف) $y = |2x-2| - |x-3|$

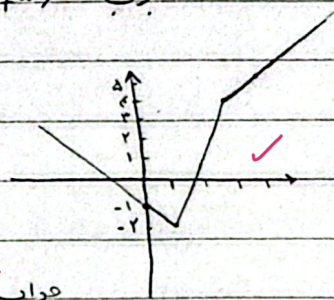
$2x-2=0 \Rightarrow x=1 \quad y=-2$

$x-3=0 \Rightarrow x=3 \quad y=2$

$x=0 \quad y=-1$

$x=2 \quad y=0$

$R_y = [-2, +\infty)$ جواب



سوال شصت

(2)

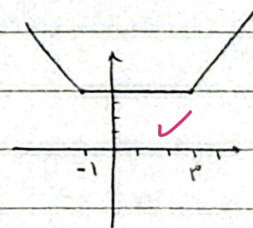
10

ب) $y = |x-2| + |x+1|$

طاني $x=2 \quad y=4 \quad x \geq 2$

$x=-1 \quad y=2 \quad x \leq -1$

$R_y = [2, +\infty)$ جواب



15

$|x-2| - |x+2| \leq 1 \Rightarrow R = (-3, -\frac{1}{3})$ جواب

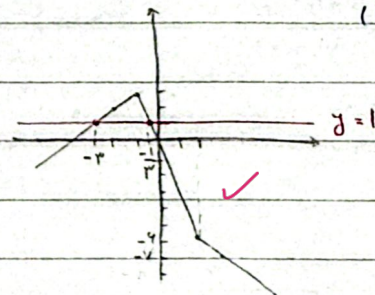
$x=2 \Rightarrow y=-4$

$x=-1 \Rightarrow y=3$

$x=3 \Rightarrow y=-7 \quad x=-2 \Rightarrow y=2$

$x \leq -1 \Rightarrow x+2=1 \Rightarrow x=-3$

$-1 < x < 2 \Rightarrow -x-2=1 \Rightarrow x=-\frac{1}{3}$



سوال هفت

(2)

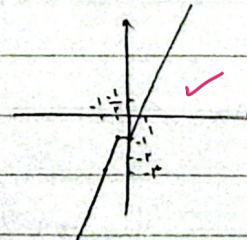
20

$y = 2x - |2x+1| + |x| \Rightarrow R_y = \mathbb{R}$ جواب

$x = -\frac{1}{2} \Rightarrow y = -1$

$x=0 \Rightarrow y=-1$

$x=-1 \Rightarrow y=-3 \quad x=1 \Rightarrow y=1$



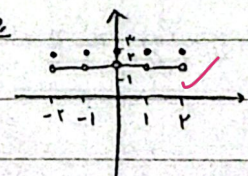
سوال هفت

(2)

25

الف) $y = [x] + [3-x] \Rightarrow y = [x] + [-x] + 3 \Rightarrow \begin{cases} 3 & x \in \mathbb{Z} \\ 2 & x \notin \mathbb{Z} \end{cases}$
 * $\begin{cases} 0 & x \in \mathbb{Z} \\ -1 & x \notin \mathbb{Z} \end{cases}$

$R_y = \{2, 3\}$ جواب



سوال نین

(2)

30

1. $y = 3x - [x]$ جواب $Ry = [-4, 5]$

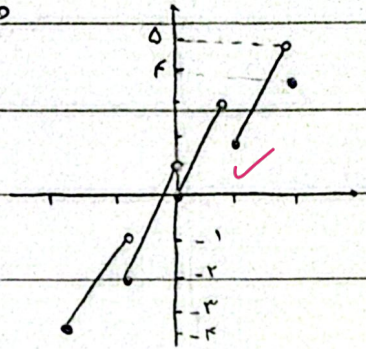
2. $-2 \leq x < -1$ $[x] = -2$, $3x + 2$

3. $-1 \leq x < 0$ $[x] = -1$, $3x + 1$

4. $0 \leq x < 1$ $[x] = 0$, $3x$

5. $1 \leq x < 2$ $[x] = 1$, $3x - 1$

6. $x = 2$ $[x] = 2$, $3x - 2 \Rightarrow y = 4$



8. الف) $y = \sin^2 x - \sin x + 1$

سوال (د)

9. $\sin x$ مقدار $= 1 \Rightarrow y = 1$

جواب $Ry = [\frac{3}{4}, 3]$

(2)

10. $\sin x$ مقدار $= -1 \Rightarrow y = 3$

11. $\frac{-b}{2a} = \frac{1}{2} \Rightarrow y = \frac{3}{4}$

13. ب) $\sin^2 x + \sin^2 x + 1$

14. $\sin^2 x$ مقدار $= 1 \Rightarrow y = 3$

جواب $Ry = [1, 3]$

15. $\sin^2 x$ مقدار $= 0 \Rightarrow y = 1$

16. $\frac{-b}{2a} = \frac{-1}{2} \Rightarrow \sin^2 x = \frac{-1}{2}$ غیر قابل قبول

