

14, 25

الف) $y = \frac{|x|+1}{|x|-1} \Rightarrow y|x| - y = |x|+1$

$y|x| - |x| = 1+y \Rightarrow |x|(y-1) = 1+y$

$|x| = \frac{1+y}{y-1} \Rightarrow \frac{1+y}{y-1} > 0 \Rightarrow \frac{-1}{+} \frac{1}{-} \frac{1}{+}$

$R_f = (-\infty, -1] \cup (1, +\infty)$

ب) $y = \frac{x^2+1}{|x|} \Rightarrow |x| + \frac{1}{|x|}$

$x > 0 \rightarrow 1 + \frac{1}{x} = (2)$

$R_f = [2, +\infty)$

$y = \frac{x^2+bx-r}{x^2+bx} \Rightarrow yx^2+by = x^2+bx-r$

$\Rightarrow x^2+bx-yx^2-r-by = 0 \Rightarrow x^2+x(r-y)-(r+by) = 0 \Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a}$

$x = \frac{-(r-y) \pm \sqrt{(r-y)^2 - 4(-r-by)}}{2} \Rightarrow (r-y)^2 - 4(-r-by) > 0 \Rightarrow y^2 + \epsilon - \epsilon y + 4r + 4by > 0$

$y^2 - \epsilon y + 4\epsilon > 0 \Rightarrow \Delta = 16 - 4(\epsilon^2) < 0 \Rightarrow (I) + \infty$

$R_y = \mathbb{R}$

صفحه آخر...

$f(x) = \frac{x^2-x+1/2}{x^2-x+1} \Rightarrow x^2-x+1 \neq 0 \Rightarrow \Delta = 1-2 < 0 \Rightarrow +\infty$

$D_f = \mathbb{R}$

$\frac{(x-1/2)^2}{(x-1/2)^2 + 3/4} \xrightarrow{(x-1/2)^2 = t} \frac{t}{t+3/4} \Rightarrow y = \frac{t}{t+3/4}$

$\downarrow$
تغییر متغیر
بسیار ساده است

$R_f = [0, 1)$

الف) $y = \sqrt{-x^2+4x-1}$

$\begin{cases} \frac{-b}{2a} \Rightarrow \frac{-4}{-2} = 2 \\ \frac{-b}{2a} \Rightarrow \frac{-4}{-2} = 2 \end{cases}$

$y_{max} = -1+4-1 = 2 \Rightarrow (2)$

$R_y = \sqrt{[-\infty, 4]} \Rightarrow [0, 2]$

محدوده مقادیر

ب) $y = \sqrt[3]{x^2-1}$

$\begin{cases} \frac{-b}{2a} = \frac{-1}{1} = -1 \\ y_{min} = 1-1 = 0 \end{cases}$

$y_{min} = 1-1 = 0 \Rightarrow (1)$

$R_y = \sqrt[3]{[-1, +\infty)} \Rightarrow [-1, +\infty)$

الف) $y = \log(x^2-x+1)$

$R_y = \log(\mathbb{R}) \Rightarrow R_y = (-\infty, +\infty)$

ب) $y = \log_r\left(\frac{x^2+1}{x}\right) \Rightarrow y \Rightarrow yx = x^2+1$

$x^2-yx+1 = 0 \Rightarrow \Delta = y^2-4 > 0 \Rightarrow y > 2 \text{ or } y < -2$

$y > 2, y < -2 \Rightarrow (-\infty, -2] \cup [2, +\infty)$

$R_y = \log_r(-\infty, -2] \cup [2, +\infty)$

$R_y = (-\infty, -2] \cup [2, +\infty)$

محدوده مقادیر

ب) $\frac{x^2+1}{x} = x + \frac{1}{x} \Rightarrow R = (-\infty, -2] \cup [2, +\infty)$

$\log_r \Rightarrow R_y = [1, +\infty)$

الف) $y = |x-2| - |x+1|$

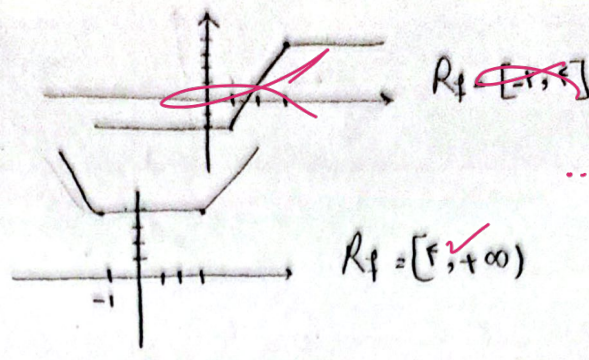
$x=2 \rightarrow y=0$

$x=-1 \rightarrow y=0$

ب) $y = |x-2| + |x+1|$

$x=2 \rightarrow y=4$

$x=-1 \rightarrow y=4$



$R_f = [-1, 2]$

$R_f = [2, +\infty)$

$|x-2| - |x+1| \leq 1$

$x < -1$	$-1 \leq x < 2$	$x \geq 2$
$2-x + x+1 = 3$	$2-x - (x+1) = 1-2x$	$x-2 - (x+1) = -3$
$3 \leq 1$ (False)	$1-2x \leq 1 \rightarrow -2x \leq 0 \rightarrow x \geq 0$	$-3 \leq 1$ (True)
(I)	(II)	(III)

(I) $x < -1 \rightarrow x \leq -1 \rightarrow (-\infty, -1]$

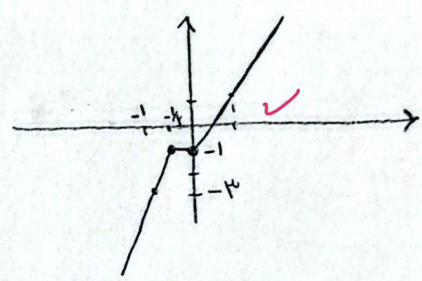
(II) $-1 \leq x < 2 \rightarrow x \geq 0 \rightarrow [0, 2)$

(III) $x \geq 2 \rightarrow x \leq 2 \rightarrow [2, +\infty)$

$(-\infty, -1] \cup [0, +\infty)$

$y = 2x - |x+1| + |x|$

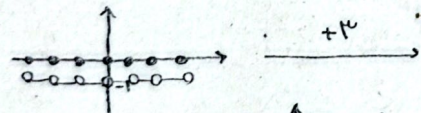
$x < -1$	$-1 \leq x < 0$	$x \geq 0$
$2x - (x+1) + (-x) = -1$	$2x - (x+1) + x = x-1$	$2x - (x+1) + x = x-1$
$-1 \leq -1$ (True)	$x-1 \leq -1 \rightarrow x \leq 0$	$x-1 \leq -1 \rightarrow x \leq 0$
(I)	(II)	(III)



$R_f = \mathbb{R}$

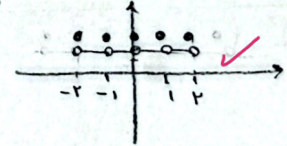
الف) $y = [x] + [x-1] \rightarrow [x] + [-x] + 1$

$[x] + [-x] \leq 0$



نقطه تقاطع خط با دامنه $[2, 2]$ و دامنه $[2, 2]$ و دامنه $[2, 2]$

$[x] + [-x] + 1$



$R_y = \{2, 3\}$

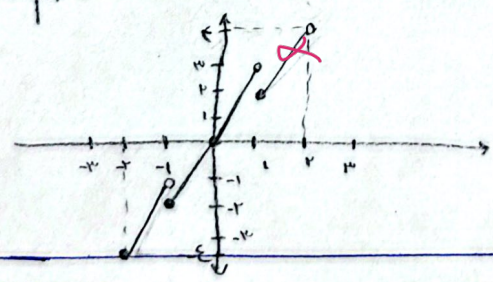
$[x-1] \rightarrow [x] = 2$

ب) $y = 2x - [x]$

$[0, 1) \rightarrow [x] = 0$

$[1, 2) \rightarrow [x] = 1$

$[-1, 0) \rightarrow [x] = -1$



صفر به ...

ج) $y = \sin^2 x - \sin x + 1$

$\sin x = -1 \rightarrow y = 1$

$\sin x = 1 \rightarrow y = 1$

$\sin x = \frac{1}{2} \rightarrow y = \frac{3}{4}$

$R_y = [\frac{3}{4}, 1]$

د) $y = \sin^2 x + \sin x + 1$

$\sin^2 x = 0 \rightarrow y = 1$

$\sin^2 x = 1 \rightarrow y = 2$

$\sin^2 x = \frac{1}{4} \rightarrow y = \frac{5}{4}$

$R_y = [\frac{5}{4}, 2]$

محدوده مقرونه

-2

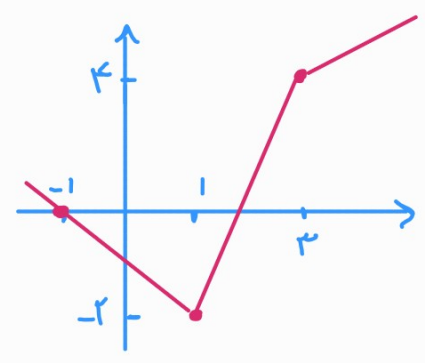
طريقين وطريق

$$y^2 + 3y = x^2 + 2x - 2 \rightarrow x^2 + (2-y)x - 2 - 3y = 0$$

$$x = \frac{y-2 \pm \sqrt{(2-y)^2 - 4(-2-3y)}}{2} = \frac{y-2 \pm \sqrt{y^2 + 14y + 12}}{2}$$

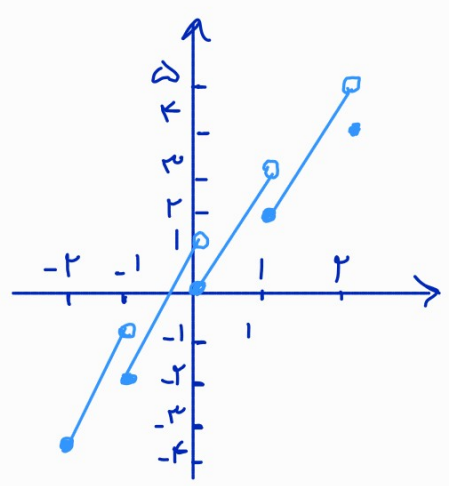
$$\Delta \geq 0 \rightarrow y^2 + 14y + 12 \geq 0 \rightarrow \frac{-4}{+} \frac{-2}{-} \quad R_y = (-\infty, -4] \cup [-2, +\infty)$$

الف $y = \begin{cases} x+1 & x \geq 3 \\ 3x-5 & 1 \leq x \leq 3 \\ -x-1 & x \leq 1 \end{cases}$



$$\Rightarrow R_y = [-2, +\infty)$$

-4



$$R_y = [-2, \infty)$$

-4